Soln: 9

$$\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$$

Put $x = \tan\theta$

$$\frac{dx}{d\theta} = \frac{1}{\cos^2\theta}$$

$$dx = \frac{d\theta}{\cos^2\theta}$$

$$\therefore \int \frac{1}{\tan^2\theta\sqrt{1+\tan^2\theta}} \cdot \frac{1}{\cos^2\theta}\,d\theta$$

$$\Rightarrow \int \frac{1}{\frac{\sin^2\theta}{\cos^2\theta}\cdot\sqrt{\frac{1}{\cos^2\theta}}} \cdot \frac{1}{\cos^2\theta}\,d\theta$$

$$\Rightarrow \int \frac{\cos\theta}{\sin^2\theta}\,d\theta$$

Put $t = \sin\theta$

$dt = \cos\theta\,d\theta$

$$\therefore \Rightarrow \int \frac{dt}{t^2} \Rightarrow \int t^{-2}\,dt \Rightarrow \frac{t^{-2+1}}{-2+1} + C \Rightarrow \frac{t^{-1}}{-1} + C$$

$$\therefore \Rightarrow \frac{-1}{t} \Rightarrow \frac{-1}{\sin\theta} \Rightarrow \frac{-1}{\sin(\tan^{-1}x)}$$

$1 + \tan^2\theta$

$\Rightarrow 1 + \frac{\sin^2\theta}{\cos^2\theta}$

$\Rightarrow \frac{\cos^2\theta}{\cos^2\theta} + \frac{\sin^2\theta}{\cos^2\theta}$

$\frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}$

$\Rightarrow \frac{1}{\cos^2\theta}$

$$\Rightarrow \frac{-\frac{1}{x}}{\frac{x}{\sqrt{1+x^2}}} = -\frac{\sqrt{1+x^2}}{x} + C \quad \text{Ans}$$