Solⁿ. 12 :-

$$I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)}\, dx$$

Let $\sin x = t$

differentiate both side w.r.t x

$$\frac{dt}{dx} = \cos x$$

$$dt = \cos x\, dx$$

$$\therefore \int \frac{dt}{(1+t)(2+t)}$$

By using fractional method

$$\frac{1}{(1+t)(2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)}$$

$$1 = (2+t)A + B(1+t)$$

$$1+t = 0$$

Put, $t = -1$

$A = 1$, $B = 2$

$$\int \frac{1}{(1+t)(2+t)}\, dt = \int \frac{1}{(1+t)}\, dt + \int \frac{2}{(2+t)}\, dt$$

$$= \log(1+t) + 2\log(2+t) + c$$

$$= \log(1+\sin x) + 2\log(2+\sin x) + c$$