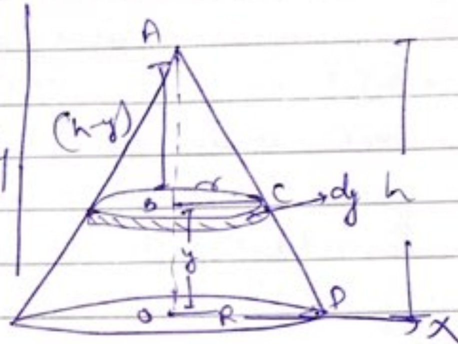
Answer - 9 :-

∴ Expression for Centre of Gravity of Cone :-



Vol. of elemental disc = $dv = \pi r^2 . dy$.

$dv = \pi r^2 \left(1 - \frac{y}{h}\right)^2 dy$

for similar ABC & AOD :-

$\frac{r}{h-y} = \frac{R}{h} = r = \frac{R}{h}(h-y)$

$r = R\left(1 - \frac{y}{h}\right)$

$\bar{x} = \frac{\int x\, dv}{\int dv}$, $\bar{y} = \int \frac{y\, dv}{dv}$.

Total. volu of Cone :-

$V = \int dv = \int_0^h \pi R^2 \left(1 - \frac{y}{h}\right)^2 dy$.

$\int dv = \pi R^2 \int_0^h \left(1 + \frac{y^2}{h^2} - 2\frac{y}{h}\right) dy$.

$\int dv = \pi R^2 \left[y + \frac{y^3}{3h^2} - \frac{2y^2}{2h}\right]_0^h$

$\int dv = \pi R^2 \left[h + \frac{\cancel{h^3}h}{3\cancel{h^2}} - \frac{\cancel{2}}{\cancel{2}}\frac{\cancel{h^2}h}{\cancel{h}}\right]$

$\int dv = \pi R^2 \left[\cancel{h} + \frac{h}{3} - \cancel{h}\right]$

$\boxed{\int dv = V = \frac{1}{3}\pi R^2 h}$

Moment of V. of elemental disc about OX-Axis:

$$y\,dv = y\pi R^2\left(1-\frac{y}{h}\right)^2 dy.$$

Moment of V. of Cone about OX-Axis:-

$$\int y\,dv = \int_0^h \pi R^2\left(y+\frac{y^3}{h^2}-\frac{2y^2}{h}\right)dy.$$

$$\int y\,dv = \pi R^2\left[\frac{y^2}{2}+\frac{y^4}{4h^2}-\frac{2y^3}{3h}\right]_0^h$$

$$\int y\,dv = \pi R^2\left[\frac{h^2}{2}+\frac{h^{\cancel{4}2}}{4\cancel{h^2}}-\frac{2h^{\cancel{3}2}}{3\cancel{h}}\right]$$

$$\int y\,dv = \pi R^2\left[\frac{6h^2+3h^2-8h^2}{12}\right]$$

$$\int y\,dv = \frac{\pi R^2h^2}{12}$$

$$\bar{V} = \frac{\frac{\cancel{\pi R^2}h^{\cancel{2}}}{\cancel{12}\,4}}{\frac{1}{\cancel{3}}\cancel{\pi R^2 h}} = \boxed{\bar{V} = \frac{h}{4}}.$$

$$\boxed{\begin{array}{l}\text{Centre of Gravity of Cone from Base} = h/4. \\ \text{" " " " " " Apex} = \frac{3h}{4} \quad \left\{h-\frac{h}{4}\right\}.\end{array}}$$

Proved.