Ans 2 $\int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

$I = \int \frac{\cos x}{(1+\sin x)(2+\sin x)} dx$

$\sin x = t$

$\frac{dt}{dx} = \cos x \Rightarrow dx = \frac{dt}{\cos x}$

$I = \int \frac{dt}{(1+t)(2+t)}$

$\frac{1}{(1+t)(2+t)}$

$\frac{1}{(1+t)(2+t)} = \frac{A}{1+t} + \frac{B}{2+t}$

$A(2+t) + B(1+t) = 1$

$t(A+B) + 1(2A+B) = 1$

$t \cdot (A+B) + 1(2A+B) = 1$

$A+B = 0 \quad -(i) \cdot 2 \quad A+B = 1 \quad -(ii)$

$2A + B = 1$

$-A - B = 0$

$\boxed{A = 1} \cdot \boxed{B = -1}$

$I = \int \left(\frac{1}{t+1} - \frac{t}{t-2} \right) dt$

$I = \log(t+1) - \log(t+2) + C$

$I = \log \left| \frac{t}{t+2} \pm 1 \right| + C$

$\boxed{I = \log \left| \frac{\sin x + 1}{\sin x + 2} \right| + C}$