Q12) $\int \frac{\cos x}{(1+\sin x)(2+\sin)} dx$

Let $\sin x = t$

$\cos x \, dx = dt$

$\int \frac{dt}{(1+t)(2+t)}$

By using fractional method.

$$\frac{1}{(1+t)(2+t)} = \frac{A}{(1+t)} + \frac{B}{(2+t)}$$

$$1 = (2+t)A + B(1+t)$$

$1 + t = 0$

Put, $t = -1$

$A = 1$

$t = -2$

$B = 2$

$$\int \frac{1}{(1+t)(2+t)} dt = \int \frac{1}{1+t} dt + \int \frac{2}{2+t} dt$$

$$= \log(1+t) + 2\log(2+t) + C$$

$$= \log(1+\sin x) + 2\log(2+\sin x) + C$$