$$I = \int \frac{1}{x^2\sqrt{1+x^2}}\,dx$$

$$= \int \frac{dx}{x^3\sqrt{\frac{1}{x^2}+1}}$$

$$\frac{1}{x^2}+1 = t$$

$$\frac{-2}{x^3} = \frac{dt}{dx}$$

$$\frac{dx}{x^3} = \frac{dt}{-2}$$

$$\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\int t^{-1/2}\,dt$$

$$= -\frac{1}{2}\left[\frac{t^{1/2}}{1/2}\right] + c$$

$$= -\sqrt{t} + c$$

$$= -\sqrt{\frac{1}{x^2}+1} + c$$