Find $\int \frac{1}{x^2\sqrt{1+x^2}}\,dx$

$$\int \frac{dx}{x^2\sqrt{1+x^2}}$$

$x = \tan\theta$ $\qquad dx = \sec^2\theta\,d\theta$

$x^2 = \tan^2\theta$

$1 + x^2 = 1 + \tan^2\theta = \sec^2\theta$.

$$\int \frac{\sec^2\theta\,d\theta}{\tan^2\theta\sqrt{\sec^2\theta}}$$

$$= \int \frac{\sec\theta\,d\theta}{\tan^2\theta}$$

$$= \int \frac{\frac{1}{\cos\theta}\,d\theta}{\frac{\sin^2\theta}{\cos^2\theta}}$$

$$= \int \frac{\cos\theta\,d\theta}{\sin^2\theta}$$

$$= \int \cot\theta\,\operatorname{cosec}\theta\,d\theta$$

$$= -\operatorname{cosec}\theta + c$$

$\operatorname{cosec}^2\theta = 1 + \cot\theta$

$\operatorname{cosec}^2\theta = 1 + \frac{1}{\tan^2\theta} \Rightarrow \operatorname{cosec}\theta = \frac{\sqrt{1+1}}{\tan^2\theta}$

$$= \sqrt{1 + \frac{1}{x^2} - 1}\ \sqrt{\frac{x^2+1}{x}}$$

$$= -\sqrt{\frac{x^2+1}{x}} + c \quad \text{Ans}$$