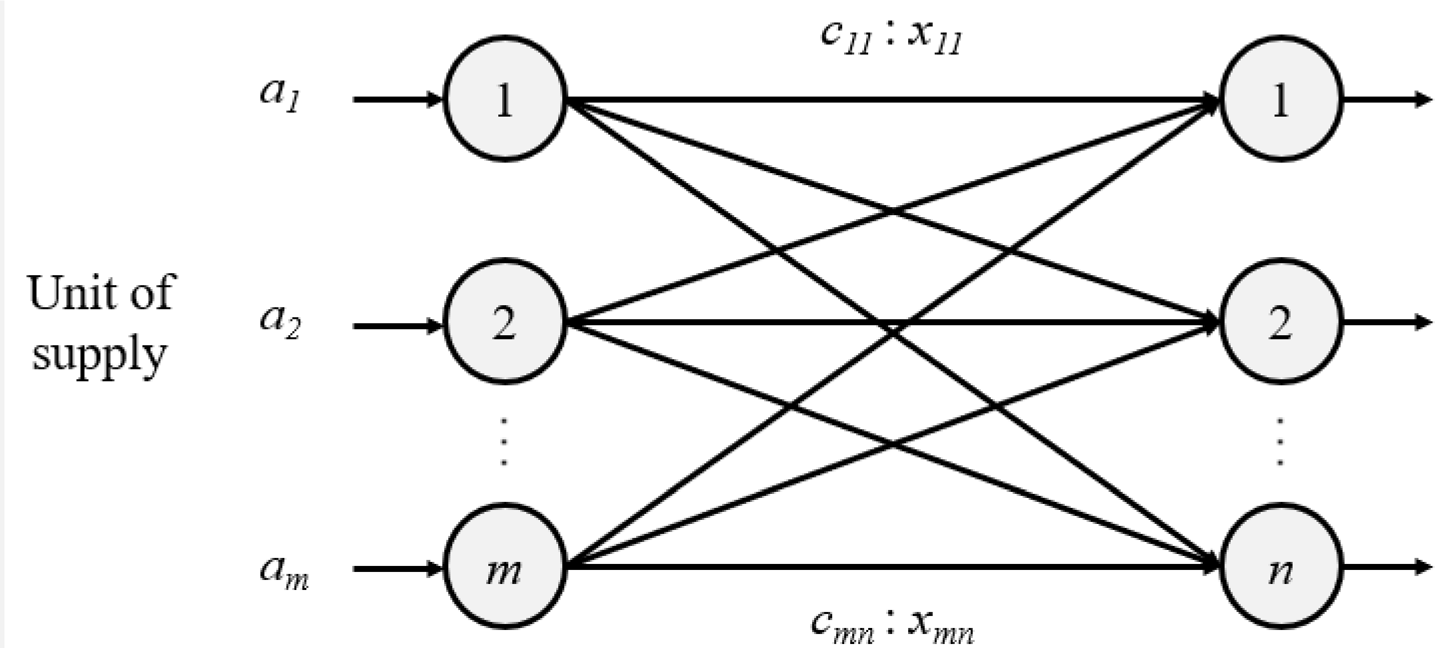
Transportation Problem:

The transportation problem is a special type of linear programming problem where the objetive consists in **minimizing transportation cost** of a given commodity from a number of **sources** or **origins** (e.g. factory, manufacturing facility) to a number of **destinations** (e.g. warehouse, store). Each source has a limited supply (i.e. maximum number of products that can be sent from it) while each destination has a demand to be satisfied (i.e. minimum number of products that need to be shipped to it). The cost of shipping from a source to a destination is directly proportional to the number of units shipped.

Basic Notation:

- m = number of sources ($i = 1 \ldots m$)
- n = number of destinations ($j = 1 \ldots n$)
- $c_{i,j}$ = unit cost of shipping from source i to destination j
- $x_{i,j}$ = amount shipped from source i to destination j
- a_i = supply at source i
- b_j = demand at destination j



LP Transportation Problem Diagram

Sources are represented by rows while destinations are represented by columns. In general, a transportation problem has *m* rows and *n* columns. The problem is solvable if there are exactly (*m*+*n*-1) basic variables.

	Destination 1	***Destination 2***	***Destination n***
Supplier 1	*c11* *x11*	*c12* *x12*	*c1n* *x1n*
Supplier 2	*c21* *x21*	*c22* *x22*	*c2n* *x2n*
Supplier m	*cm1* *xm1*	*cm2* *xm2*	*cmn* *xmn*
Demand	*b1*	*b2*	*bn*

LP Transportation Problem Simplex Tableau

Types of Transportation Problems

There are two different types of transportation problems based on the initial given information:

- **Balanced Transportation Problems:** cases where the total supply is equal to the total demand.
- **Unbalanced Transportation Problems:** cases where the total supply is not equal to the total demand. When the supply is higher than the demand, a *dummy* destination is introduced in the equation to make it equal to the supply (with shipping costs of $0); the excess supply is assumed to go to inventory. On the other hand, when the demand is higher than the supply, a *dummy* source is introduced in the equation to make it equal to the demand (in these cases there is usually a penalty cost associated for not fulfilling the demand).

In order to proceed with the solution of any given transportation problem, the first step consists in verifying if it is balanced. If it is not, it must be balanced accordingly.

The *lpSolve* package from R contains specific functions for solving linear programming transportation problems. For the following example, let's consider the following mathematical model to be solved:

$$\min z = \sum_{i=1}^{3} \sum_{j=1}^{4} c_{ij} x_{ij}$$

s.t.

$$x_{11} + x_{12} + x_{13} + x_{14} \leq 15$$
$$x_{21} + x_{22} + x_{23} + x_{24} \leq 25$$
$$x_{31} + x_{32} + x_{33} + x_{34} \leq 10$$
$$x_{11} + x_{21} + x_{31} \geq 5$$
$$x_{12} + x_{22} + x_{32} \geq 15$$
$$x_{13} + x_{23} + x_{33} \geq 15$$
$$x_{14} + x_{24} + x_{34} \geq 15$$
$$x_{ij} \geq 0$$

LP Transportation Problem — Mathematical Model

	Customer 1	Customer 2	Customer 3	Customer 4	Supply
Supplier 1	10	2	20	11	15
Supplier 2	12	7	9	20	25
Supplier m	4	14	16	18	10
Demand	5	15	15	15	

LP Transportation Problem — Simplex Tableau

An assignment problem can be viewed as a special case of a transportation problem. In a transportation model, sources and destinations are present; in an assignment model, there are facilities, and jobs which have to be assigned to those facilities. Unlike a transportation model, in an assignment model, number of facilities (sources) is equal to number of jobs (destinations).

However, the transportation algorithm is not useful while dealing with assignment problems. In an assignment problem, when an assignment is made, the row as well as column requirements are satisfied simultaneously, resulting in degeneracy. This occurs since only one assignment is allowed per row and column. Thus, the assignment model is a completely degenerate form of the transportation model.