Section (6) I

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Ques (1)

Ans
$$x^2\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$$

Let $x^2\dfrac{d^2y}{dx^2} = D(D-1)y$ $\qquad x = e^z$

$\qquad z = \log x$

$$4x\frac{dy}{dx} = 4Dy$$

$$D(D-1)y + 4Dy + 2y = e^{x}$$

$$(D^2+3D+2)y = e^{e^z}$$

A.E. $= m^2+3m+2=0$

$m^2+2m+m+2=0$

$m(m+2)+1(m+2)=0$

$(m+2)(m+1)=0$

$m = -2, -1$

$$\boxed{CF = c_1e^{-2z} + c_2e^{-z}}$$

$$PI = \frac{1}{D^2+3D+2}e^{e^z}$$

$$PI = \frac{1}{(D+1)(D+2)}e^{e^z} \Rightarrow \left(\frac{1}{(D+1)} - \frac{1}{(D+2)}\right)e^{e^z}$$

$$\frac{1}{(D+1)} e^{e^z} - \frac{1}{D+2} e^{e^z}$$

$$= e^{-z}\int e^{z} e^{e^z}\, dz - e^{-2z}\int e^{2z} e^{e^z}\, dz$$

$$e^z = t \Rightarrow e^z\, dz \cancel{= e^{-2z}\int e^{2z} e^{e^z}\, dz}$$

$$= dt$$

$$e^{-z}\int e^{t}\, dt - e^{-2z}\int t e^{t}\, dt$$

$$\Rightarrow e^{-z} e^{t} - e^{-2z}(t e^{t} - e^{t})$$

$$e^{-z} e^{e^z} - e^{-2z}(e^{z} e^{e^z} - e^{e^z}) = e^{-2z} e^{e^z}$$

Complete solution, $y = CF + PI$

$$y = c_1 e^{-z} + c_2 e^{-2z} + e^{-2z} e^{e^z}$$

$$\boxed{y = c_1\left(1/x\right) + c_2\left(1/x^2\right) + \left(\frac{1}{x^2}\right) e^{x}}$$