Page :
Date: / /

Section – 4

Q Qus 2

Ans $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2 - 8)y = x^2 e^{\frac{1}{2}x^2}$

on comparison with $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$

$P = 2x, \quad Q = x^2 - 8, \quad R = x^2 e^{-x^2/2}$

$V = e^{-\frac{1}{2}\int P dx} = e^{-\frac{1}{2}\int 2x dx} \Rightarrow e^{-x^2/2}$

We know that, u is given by

$$\frac{d^2u}{dx^2} + Q_1 u = R_1 \quad \longleftarrow (1)$$

$$Q_1 = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4} = x^2 - 8 - \frac{1}{2}(2) - \frac{4x^2}{4}$$

$$Q_1 = -9$$

$$R_1 = \frac{R}{V} = \frac{x^2 e^{-x^2/2}}{e^{-x^2/2}} = x^2$$

on putting the value of Q_1 and R_1 in eq^n (1)

$$\frac{d^2u}{dx^2} - 9u = x^2$$

$$(D^2 - 9)u = x^2$$

(2)

Page :
Date: / /

Auxillary eq^n, $m^2 - 9 = 0$

$$m = \pm 3$$

$$\boxed{CF = C_1 e^{3x} + C_2 e^{-3x}}$$

$$PI = \frac{1}{D^2 - 9} x^2 = \frac{1}{9}\left(1 - \frac{D^2}{9}\right) x^2$$

$$PI = \frac{1}{9}\left(1 + \frac{D^2}{9}\right) x^2 \Rightarrow \frac{1}{9}\left(x^2 + \frac{2}{9}\right)$$

Complete solution

$$y = CF + PI = C_1 e^{3x} + C_2 e^{3x} + \frac{1}{9}\left(x^2 + \frac{2}{9}\right)$$

$$y = ux\left[C_1 e^{3x} + C_2 e^{-3x} - \frac{1}{9}\left(x^2 + \frac{2}{9}\right)\right] e^{x/2}$$