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Section → A

Ques → 2

Ans → $f(x) = x - x^2, \quad 0 < x < 1$

$f(x) = x - x^2, \quad 0 < x < 1$

$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\frac{n\pi x}{1}$

$a_0 = 2\int_0^1 f(x)\,dx = 2\int_0^1 (x - x^2)\,dx$

$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{2}{6} = \frac{1}{3}$

$a_n = \frac{2}{1}\int_0^1 f(x)\cos\frac{n\pi x}{1}\,dx = 2\int_0^1 (x - x^2)\cos n\pi x$

$= 2\left[(x - x^2)\left(\frac{\sin n\pi x}{n\pi}\right) - (1-2x)\left(-\frac{\cos n\pi x}{n^2\pi^2}\right) + (-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$

$= 2\left[(-1)\frac{\cos n\pi}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right] = 2\left[\frac{(-1)^{n+1}}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right]$

$$\boxed{f(x) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left[(-1)^{n+1} - 1\right]\cos n\pi x}$$

Ans