Sec - 7

1. Ans

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi t}{l}$$

$$a_0 = \frac{2}{l}\int_0^{1} f(t)\,dt = \frac{2}{2}\left[\int_0^1 2t\,dt + \int_1^2 2(2-t)\,dt\right]$$

$$= \left[\left(\frac{2t^2}{2}\right)_0^1 + \left(4t - t^2\right)_1^2\right]$$

$$a_0 = [1+1] = 2$$

$$a_n = \frac{2}{2}\int_0^1 f(t) \cos \frac{n\pi t}{l}\,dt$$

$$= \frac{2}{2}\left[\int_0^1 2t \cos \frac{n\pi t}{2}\,dt + \int_1^2 2(2-t)\cos\frac{n\pi t}{2}\,dt\right]$$

using integration by parts

$$= \frac{2}{2}\left[\left(2t\frac{2}{n\pi}\sin\frac{n\pi t}{2} + 2\frac{2^2}{n^2\pi^2}\cos\frac{n\pi t}{2}\right)_0^1 + \left(2(2-t)\frac{2}{n\pi}\sin\frac{n\pi t}{2} - 2\frac{2^2}{n^2\pi^2}\cos\frac{n\pi t}{2}\right)_1^2\right]$$

$$= \left[\left(\frac{4}{n\pi}\sin\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\left(\cos\frac{n\pi}{2} - 1\right)\right) + \left(-\frac{8}{n^2\pi^2}\cos n\pi - \frac{4}{n\pi}\sin\frac{n\pi}{2} + \frac{8}{n^2\pi^2}\cos\frac{n\pi}{2}\right)\right]$$

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$$= \frac{16}{n^2\pi^2}\cos\frac{n\pi}{2} - \frac{8}{n^2\pi^2} - \frac{8}{n^2\pi^2}\cos n\pi$$

$$= \frac{8}{n^2\pi^2}\left[2\cos\frac{n\pi}{2} - 1 - \cos n\pi\right]$$

When n is odd, $\cos\frac{n\pi}{2} = 0$ and $\cos n\pi = -1$

$\therefore a_n = 0 \Rightarrow a_1 = a_3 = a_5 \cdots\cdots = 0$

When n is even.

$$a_2 = \frac{8}{2^2\pi^2}\left[2\cos\frac{2\pi}{2} - 1 - \cos 2\pi\right] = -\frac{8}{\pi^2}$$

$$a_4 = \frac{8}{4^2\pi^2}\left[2\cos\frac{4\pi}{2} - 1 - \cos 4\pi\right] = 0$$

$$a_6 = \frac{8}{6^2\pi^2}\left[2\cos\frac{6\pi}{2} - 1 - \cos 6\pi\right] = -\frac{8}{9\pi^2}$$

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\cos\frac{n\pi t}{l}$$

$$= 1 + \left(-\frac{8}{\pi^2} + 0 - \frac{8}{9\pi^2}\right) = 1 - \frac{8}{\pi^2}\left(1 + \frac{1}{9} + \cdots\right)$$