Sec → 4

2 Ans $(D^3 - 3D + 2)y = x^2 + 2x + 1$

Auxiliary equation -

$$m^2 - 3m + 2 = 0$$

$$(m-1)(m-2) = 0$$

$$m = 1, 2$$

$$CF = C_1 e^x + C_2 e^{2x}$$

$$PI = \frac{1}{(D^2 - 3D + 2)}(x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 + \frac{D^2 - 3D}{2}\right]^{-1}(x^2 + 2x + 1)$$

$$= \frac{1}{2}\left[1 - \frac{D^2}{2} + \frac{3D}{2} + \left(\frac{D^2 - 3D}{2}\right)^2 \cdots\right](x^2 + 2x + 1)$$

$$= \frac{1}{2}\left(1 - \frac{D^2}{2} + \frac{3D}{2} + \frac{9D^2}{4}\right)(x^2 + 2x + 1)$$

$$= \frac{1}{2}\left(x^2 + 2x + 1 - \frac{2}{2} + \frac{3}{2}(2x + 2) + \frac{9}{4} \times 2\right)$$

$$= \frac{1}{2}\left[x^2 + 2x + 3x + 3 + \frac{9}{2}\right]$$

$$= \frac{1}{2}\left[x^2 + 5x + \frac{15}{2}\right]$$

$$y = C.f + P.I$$

$$= C_1 e^x + C_2 e^{2x} + \frac{1}{2}\left[x^2 + 5x + \frac{15}{2}\right]$$