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1Ans $\iiint_V x^{l-1} y^{m-1} z^{n-1}\, dx\,dy\,dz$

$$= \frac{\Gamma l\, \Gamma m\, \Gamma n}{\Gamma l+m+n+1}$$

where D is the domain $x \geq 0, y \geq 0, z \geq 0$ and $x+y+z \leq 1$

$y+z \leq 1-x = a$ (let)

Therefore given Integral becomes.

$$\int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^{l-1} y^{m-1} z^{n-1}\, dz\,dy\,dx$$

$$= \int_0^1 x^{l-1} \left[\int_0^a \int_0^{a-y} y^{m-1} z^{n-1}\, dz\,dy \right] dx$$

$$= \int_0^1 x^{l-1} \frac{\Gamma m\, \Gamma n}{\Gamma m+n+1} a^{m+n}\, dx$$

$$= \frac{\Gamma m\, \Gamma n}{\Gamma m+n+1} \int_0^1 x^{l-1} (1-x)^{m+n+1-1}\, dx$$

$$= \frac{\Gamma m \, \Gamma n}{\Gamma(m+n+1)} \, \beta(l, m+n+1)$$

$$= \frac{\Gamma m \, \Gamma n}{\Gamma(m+n+1)} \cdot \frac{\Gamma l \, \Gamma(m+n+1)}{\Gamma(l+m+n+1)}$$

$$= \frac{\Gamma l \, \Gamma m \, \Gamma n}{\Gamma(l+m+n+1)}$$