Sec → 4

2 Ans $\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2-8)y = x^2 e^{-x^2/2}$

On comparison with, $\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$ we have.

$P = 2x$, $Q = x^2 - 8$, $R = x^2 e^{-x^2/2}$

$u = e^{-\frac{1}{2}\int P dx} = e^{-\frac{1}{2}\int 2x dx} = e^{-\frac{x^2}{2}}$

we know that, u is given by.

$\frac{d^2u}{dx^2} + Q_1 u = R_1 \rightarrow (1)$

where $Q_1 = Q - \frac{1}{2}\frac{dP}{dx} - \frac{P^2}{4} = x^2 - 8 - \frac{1}{2}(2) - \frac{4x^2}{4}$

$Q = -9$

$R_1 = \frac{R}{u} = \frac{x^2 e^{-x^2/2}}{e^{-x^2/2}} = x^2$

On putting the value of Q and R_1 in eqn (1) we get

$$\frac{d^2u}{dx^2} - 9u = x^2$$

$$(D^2 - 9)u = x^2$$

Auxiliary equation, $m^2 - 9 = 0$

$$m = \pm 3$$

$$Cf = c_1 e^{3x} + c_2 e^{-3x}$$

$$PI = \frac{1}{D^2 - 9} x^2 = \frac{1}{9}\left(1 - \frac{D^2}{9}\right)^{-1} x^2 = \frac{1}{9}\left(1 + \frac{D^2}{9}\right) x^2$$

$$PI = \frac{1}{9}\left(x^2 + \frac{2}{9}\right)$$

Complete solution,

$$u = Cf + PI = c_1 e^{3x} + c_2 e^{-3x} - \frac{1}{9}\left(x^2 + \frac{2}{9}\right)$$

Thus

$$y = uv = \left[c_1 e^{3x} + c_2 e^{-3x} - \frac{1}{9}\left(x^2 + \frac{2}{9}\right)\right] e^{-\frac{x}{2}}$$