Sec ⇒ 6

(1) $x^2 \frac{d^2y}{dx^2} + 4x\frac{dy}{dx} + 2y = e^x$

$\{D(D-1) + 4D + 2\}y = e^x$ $\quad [\because x = e^z)$

$(D^2 + 3D + 2)y = e^{e^z}$

Auxiliary equation,

$m^2 + 3m + 2 = 0 \Rightarrow m = -1, -2$

$cf = c_1 e^{-z} + c_2 e^{-2z}$

$PI = \frac{1}{D^2+3D+2} e^{e^z}$

(using General method to find PI)

$= \frac{1}{(D+1)(D+2)} e^{e^z} = \left(\frac{1}{D+1} - \frac{1}{D+2}\right) e^{e^z}$

$= \frac{1}{D+1} e^{e^z} - \frac{1}{D+2} e^{e^z}$

$= e^{-z}\int e^z e^{e^z} dz - e^{-2z}\int e^{2z} e^{e^z} dz$

Let $e^z = t \Rightarrow e^z = dt$

$= e^{-z}\int e^t dt - e^{-2z}\int te^t dt = e^{-z}e^t - e^{-2z}(te^t - e^t)$

$= e^{-z}e^{e^z} - e^{-2z}(e^z e^{e^z} - e^{e^z}) = e^{-2z} e^{e^z}$

Complete Solution, $y = CF + PI$

$$y = c_1 e^{-z} + c_2 e^{-2z} + e^{-2z} e^{e^{z}}$$

$$y = c_1\left(\frac{1}{x}\right) + c_2\left(\frac{1}{x^2}\right) + \left(\frac{1}{x^2}\right) e^{x}$$