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① Ans

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx = \frac{1}{\pi}\left[\int_{-\pi}^{\pi/2}(-1)\,dx + \int_{\pi/2}^{\pi}(1)\,dx\right]$$

$$= \frac{1}{\pi}\left[-\left(-\frac{\pi}{2}+\pi\right)+\left(\pi-\frac{\pi}{2}\right)\right]$$

$$a_0 = 0$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2} -\cos nx\,dx + \int_{\pi/2}^{\pi}\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[-\left\{\frac{\sin nx}{n}\right\}_{-\pi}^{-\pi/2} + \left\{\frac{\sin nx}{n}\right\}_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\sin n\pi/2}{n} - \frac{\sin n\pi}{n} + \frac{\sin n\pi}{n} - \frac{\sin n\pi/2}{n}\right]$$

$$a_n = 0$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$$

$$= \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2} -\sin nx\,dx + \int_{\pi/2}^{\pi}\sin nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left\{\frac{\cos nx}{n}\right\}_{-\pi}^{-\pi/2} - \left\{\frac{\cos nx}{n}\right\}_{\pi/2}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi/2}{n} - \frac{\cos n\pi}{n} - \frac{\cos n\pi}{n} + \frac{\cos n\pi/2}{n}\right]$$

$$b_n = \frac{2}{\pi n}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]$$

Hence required series is,

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin nx$$

Putting $x = \pi/2$ in the above series.

$$[f(x)]_{x=\pi/2} = \sum \frac{2}{\pi n}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin\frac{n\pi}{2}$$

$$\frac{0+1}{2} = \frac{2}{\pi}\sum \frac{1}{n}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin\frac{n\pi}{2}$$

Putting $n = 1, 2, 3, 4 \cdots$

$$\frac{\pi}{4} = \frac{1}{1}\left(\cos\frac{\pi}{2} - \cos\pi\right)\sin\frac{\pi}{2} + 0$$

$$+ \frac{1}{3}\left(\cos\frac{3\pi}{2} - \cos 3\pi\right)\sin\frac{3\pi}{2} + 0$$

$$+ \frac{1}{5}\left(\cos\frac{5\pi}{2} - \cos 5\pi\right)\sin\frac{5\pi}{2} + 0 + \cdots$$

$$\frac{\pi}{4} = 1 + \frac{1}{3}(-1) + \frac{1}{5}(1) + \frac{1}{7}(-1) + \cdots$$

$$\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots$$