Section :- 3

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(2) Discuss Einstein mass energy Equation $E = mc^2$

Ans Einstein mass energy :

(1) Suppose a force 'f' is acting on a body of mass 'm' in the same direction as its velocity 'v'

(2) The gain in K·E in the body is the form of work done.

(3) If a force 'f' displaces the particle through a small distance 'ds', then work done :

$$dw = dk = f \cdot ds$$

According to Newton's law of motion

$$f = \frac{dp}{dt} = \frac{d(mv)}{dt}$$

$$\Rightarrow f = \frac{m\,dv}{dt} + v\frac{dm}{dt}$$

multiplying 'ds' on both sides, we get

$$f \cdot ds = m\frac{ds}{dt} \cdot dv + v\frac{ds}{dt} \cdot dm$$

form eq $\Rightarrow dk = mv\,dv + v^2\,dm$ $\quad \because \left(\frac{ds}{dt} = v\right)$

But we know that $- \quad m = \dfrac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$

On differentiating, we get

$$dm = m_0\left(-\frac{1}{2}\right)\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}\left(\frac{-2v}{c^2}\right)dv = \frac{m_0 v\,dv}{c^2\left(1 - \frac{v^2}{c^2}\right)^{\frac{3}{2}}}$$

$$dm = \frac{mv\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)} \qquad \because \; m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

$$dm(c^2 - v^2) = mv\,dv$$

(8) Now putting the value of eq. (1.24.3) in eq 1.24.2, we get

$$dk = (c^2 - v^2)\,dm + v^2\,dm = c^2\,dm$$

(9) If the change in kinetic energy of a particle be K when it's mass changes from rest mass m_0 to relativistic mass m, than

$$\int_0^K dk = \int_{m_0}^{m} c^2\,dm$$

$$K = c^2(m - m_0) = c^2(\Delta m)$$

Total energy of particle

E = Relativistic K.E + Rest mass energy

$$E = (m - m_0)c^2 + m_0c^2 = \boxed{mc^2}$$

This is Einstein mass energy.