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$$\boxed{S = x^2} \quad \text{---} \quad (4)$$

Normal form is given by -

$$\frac{d^2V}{dx^2} + IV = S \quad \text{---} \quad (I)$$

On putting the value of I & S in eqn (I)

$$\Rightarrow \frac{d^2v}{dx^2} - 9v = x^2$$

$$\Rightarrow (D^2 - 9)v = x^2.$$

Now the A.E. is -

$$m^2 - 9 = 0$$

$$m^2 = 9$$

$$\boxed{m = \pm 3}$$

So, $\boxed{C.F. = c_1e^{3x} + c_2e^{-3x}}$

$$\Rightarrow P.I. = \frac{1}{D^2 - 9}(x^2) = \frac{1}{-9\left(1 - \frac{D^2}{9}\right)}(x^2)$$

$$\Rightarrow PI = \frac{-1}{9}\left[1 - \frac{D^2}{9}\right]^{-1}(x^2) = \frac{-1}{9}\left[1 + \frac{D^2}{9}\right](x^2)$$

$$\Rightarrow PI = \frac{-1}{9}\left[x^2 + \frac{2}{9}\right]$$

So Complete Solution, $u = C.F. + P.I.$

$$\Rightarrow u = c_1e^{3x} + c_2e^{-3x} - \frac{1}{9}\left(x^2 + \frac{2}{9}\right)$$

Thus, $y = u \times v$

$$y = \left[c_1e^{3x} + c_2e^{-3x} - \frac{1}{9}\left(x^2 + \frac{2}{9}\right)\right]e^{-x^2/2}$$

SECTION - 02

(07)

Question answer No. - 02

Givenas-

$$\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + (x^2-8)y = x^2 e^{-0.5x^2} \quad \text{— I}$$

On comparing with equation -

$$\frac{d^2y}{dx^2} + P\frac{dy}{dx} + Qy = R$$

We have,

$$\boxed{P = 2x} \quad \boxed{Q = x^2 - 8}$$

$$\boxed{R = x^2 e^{-0.5x^2} = x^2 e^{-x^2/2}}$$

$$\Rightarrow u = e^{-\frac{1}{2}\int P dx} = e^{-\frac{1}{2}\int 2x dx} = e^{-\frac{1}{2}\times 2\left(\frac{x^2}{2}\right)}$$

$$\boxed{u = e^{-x^2/2}}$$

Let $y = uv$ be the complete solution -

$$I = Q - \frac{1}{2}\frac{d}{dx}(P) - \frac{P^2}{4}$$

$$I = x^2 - 8 - \frac{1}{2}\frac{d}{dx}(2x) - \frac{(2x)^2}{4}$$

$$I = x^2 - 8 - \frac{1}{\cancel{2}}(\cancel{2}) - \frac{\cancel{4}x^2}{\cancel{4}}$$

$$I = x^2 - 8 - x^2 - 1$$

$$\boxed{I = -9}$$

$$S = \frac{R}{u} = \frac{x^2 \cancel{e^{-x^2/2}}}{\cancel{e^{-x^2/2}}}$$