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$$\Rightarrow a_n = \frac{1}{\pi} \left\{ -\left[\frac{\sin nx}{n} \right]_{-\pi}^{-\pi/2} + \left[\frac{\sin nx}{n} \right]_{\pi/2}^{\pi} \right\}$$

$$a_n = \frac{1}{\pi} \left\{ -\left[\frac{\sin(-n\pi/2) - \sin(-n\pi)}{n} \right] + \left[\frac{\sin n\pi - \sin(n\pi/2)}{n} \right] \right\}$$

$$a_n = \frac{1}{\pi} \left\{ \frac{\sin n\pi}{n} - \frac{\sin n\pi}{n} + \frac{\sin n\pi}{n} - \frac{\sin(n\pi/2)}{n} \right\}$$

$$a_n = \frac{1}{\pi} (0)$$

$$\boxed{a_n = 0}$$

$$\Rightarrow b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin nx \, dx$$

$$b_n = \frac{1}{\pi} \left\{ \int_{-\pi}^{-\pi/2} -\sin nx \, dx + \int_{\pi/2}^{\pi} \sin nx \, dx \right\}$$

$$b_n = \frac{1}{\pi} \left\{ \left[\frac{\cos nx}{n} \right]_{-\pi}^{-\pi/2} - \left[\frac{\cos nx}{n} \right]_{\pi/2}^{\pi} \right\}$$

$$b_n = \frac{1}{\pi} \left\{ \frac{\cos(-n\pi/2) - \cos(-n\pi)}{n} - \left(\frac{\cos n\pi - \cos(n\pi/2)}{n} \right) \right\}$$

$$\Rightarrow b_n = \frac{1}{\pi} \left\{ \frac{\cos n\pi/2 - \cos n\pi}{n} - \left(\frac{\cos n\pi - \cos n\pi/2}{n} \right) \right\}$$

$$\Rightarrow b_n = \frac{1}{\pi} \left\{ \frac{\cos n\pi/2 - \cos n\pi - \cos n\pi + \cos n\pi/2}{n} \right\}$$

$$\Rightarrow b_n = \frac{1}{\pi} \left\{ \frac{2 \cos n\pi/2 - 2 \cos n\pi}{n} \right\}$$

$$\Rightarrow b_n = \frac{2}{n\pi} \left\{ \cos n\pi/2 - \cos n\pi \right\}$$

Question Answer No-01

Given as:- the function -

$$f(x) = \begin{cases} -1 & -\pi < x < -0.5\pi \\ 0 & -0.5\pi < x < 0.5\pi \\ 1 & 0.5\pi < x < \pi \end{cases}$$

The fourier series for the function -

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

$$\Rightarrow a_0 = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) dx =$$

$$\Rightarrow a_0 = \frac{1}{\pi} \left[\int_{-\pi}^{-\frac{\pi}{2}} (-1) dx + \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (0) dx + \int_{\frac{\pi}{2}}^{\pi} (1) dx \right]$$

$$a_0 = \frac{1}{\pi} \left\{ -\left[x \right]_{-\pi}^{-\frac{\pi}{2}} + \left[x \right]_{\frac{\pi}{2}}^{\pi} \right\}$$

$$a_0 = \frac{1}{\pi} \left\{ -\left[-\frac{\pi}{2} + \pi \right] + \left[\pi - \frac{\pi}{2} \right] \right\}$$

$$a_0 = \frac{1}{\pi} \left\{ +\pi - \pi + \pi - \frac{\pi}{2} \right\}$$

$$a_0 = \frac{1}{\pi} (0)$$

$$\boxed{a_0 = 0}$$

$$\Rightarrow a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos nx dx$$

$$\Rightarrow a_n = \frac{1}{\pi} \left\{ \int_{-\pi}^{-\frac{\pi}{2}} -\cos nx dx + \int_{\frac{\pi}{2}}^{\pi} \cos nx dx \right\}$$