SECTION - 2

Question Answer No:- 01

Given as- $f(t) = \dfrac{e^{-t}\sin t}{t}$

let

$$L(e^{-t}\sin t) = \frac{1}{(p+1)^2+1}$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \int_p^\infty \frac{1}{(p+1)^2+1}\,dp$$

let $p+1 = x$

$$\frac{dp}{dx} = 1$$

$$\boxed{dp = dx}$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \left[\tan^{-1}\left(\frac{p+1}{1}\right)\right]_p^\infty$$

$$\left(\because \int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right)\right)$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \tan^{-1}(\infty+1) - \tan^{-1}(p+1)$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \tan^{-1}(\infty) - \tan^{-1}(p+1)$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \tan^{-1}\left(\tan\frac{\pi}{2}\right) - \tan^{-1}(p+1)$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \frac{\pi}{2} - \tan^{-1}(p+1) \qquad (\because \tan^{-1}\tan\theta = \theta)$$

$$L\left(\frac{e^{-t}\sin t}{t}\right) = \cot^{-1}(p+1) \qquad \left(\because \frac{\pi}{2} - \tan^{-1}\theta = \cot^{-1}\theta\right)$$

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Hence required series is –

$$f(x) = \frac{0}{2} + \sum_{n=1}^{\infty} (0)\cos nx + \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]\sin nx$$

Putting $x = \frac{\pi}{2}$ in above series is –

$$\Rightarrow f\left(\frac{\pi}{2}\right) = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]\sin\frac{n\pi}{2}$$

$$\Rightarrow \frac{0+1}{2} = \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{1}{n}\left[\cos\frac{n\pi}{2} - \cos n\pi\right]\sin\frac{n\pi}{2}$$

Put $n = 1, 2, 3, 4, \ldots$

$$\Rightarrow \frac{\pi}{4} = \frac{1}{1}\left[\cos\frac{\pi}{2} - \cos\pi\right]\sin\frac{\pi}{2} + \frac{1}{2}\left[\cos\pi - \cos 2\pi\right]\sin\pi + \frac{1}{3}\left[\cos\frac{3\pi}{2} - \cos 3\pi\right]\sin\frac{3\pi}{2} + \frac{1}{4}\left[\cos 2\pi - \cos 4\pi\right]\sin 2\pi + \ldots$$

$$\Rightarrow \frac{\pi}{4} = 1[0-(-1)]1 + \cancel{\tfrac{1}{2}}\,0 + \frac{1}{3}[(-1)] + 0 + \frac{1}{5}(1) + 0 + \frac{1}{7}(-1) + \ldots$$

$$\Rightarrow \frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots$$

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