SECTION-1

Question Answer No - 09

Given as- $f(x) = x - x^2$ in $0 < x < 1$

To find- Half range cosine series = ?

$\Rightarrow$ $f(x) = x - x^2$ $(0, 1)$

We know that,

Half range cosine series for the function, in interval (0, l)

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) \quad \text{--- (I)}$$

Where, $a_0 = \frac{2}{l}\int_0^l f(x)\,dx$

$$a_0 = \frac{2}{1}\int_0^1 (x-x^2)\,dx = 2\int_0^1 (x-x^2)\,dx$$

$$a_0 = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$a_0 = 2\left[\frac{1^2-0^2}{2} - \frac{1^3-0^3}{3}\right] = 2\left(\frac{1}{2} - \frac{1}{3}\right)$$

$$a_0 = 2\times\left(\frac{3-2}{6}\right) = \frac{1}{3}$$

$$\boxed{a_0 = \frac{1}{3}} \quad \text{--- (II)}$$

$$a_n = \frac{2}{l}\int_0^l f(x)\cos\left(\frac{n\pi x}{l}\right)dx$$

$$a_n = \frac{2}{1}\int_0^1 (x-x^2)\cos(n\pi x)\,dx$$

$$a_n = 2\int_0^1 x\cos n\pi x\,dx - 2\int_0^1 x^2\cos n\pi x\,dx$$

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$$\Rightarrow a_n = 2\left\{\left[\frac{x\sin(n\pi x)}{(n\pi)}\right]_0^1 + \left[\frac{\cos n\pi x}{(n\pi)^2}\right]_0^1\right\} - 2\left\{\left[\frac{x^2 \sin n\pi x}{n\pi}\right]_0^1 - \int_0^1 \frac{2x\sin n\pi x}{(n\pi)}\,dx\right.$$

$$\Rightarrow a_n = 2\left\{\frac{1\times\sin n\pi - 0\times\sin 0}{n\pi} + \frac{\cos n\pi - \cos 0}{(n\pi)^2}\right\} - 2\left\{\frac{1^2\times\sin n\pi - 0\sin 0}{n\pi} - \right.$$

$$\left.\frac{2}{n\pi}\int_0^1 x\sin(n\pi x)\,dx\right\}$$

$$\Rightarrow a_n = 2\left\{\frac{\sin n\pi}{n\pi} + \frac{\cos n\pi - 1}{(n\pi)^2}\right\} - 2\left\{\frac{\sin n\pi}{n\pi} - \frac{2}{n\pi}\int_0^1 x\sin n\pi x\,dx\right\}$$

$$\Rightarrow a_n = \frac{2\sin n\pi}{n\pi} + \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \frac{2\sin n\pi}{n\pi} + \frac{4}{n\pi}\left\{-\left[\frac{x\cos n\pi x}{n\pi}\right]_0^1 + \left[\frac{\sin(n\pi x)}{(n\pi)^2}\right]_0^1\right\}$$

$$\Rightarrow a_n = \cancel{\frac{2\sin n\pi}{n\pi}} + \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \cancel{\frac{2\sin n\pi}{n\pi}} + \frac{4}{n\pi}\left\{-\left(\frac{\cos n\pi - (\cos 0)\times 0}{n\pi}\right) + \frac{\sin n\pi - \sin 0}{(n\pi)^2}\right\}$$

$$\Rightarrow a_n = \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \frac{4\cos n\pi}{(n\pi)^2} + \frac{4}{n\pi}\,\frac{\sin n\pi}{(n\pi)^2}$$

$$\Rightarrow a_n = \frac{2\cos n\pi - 2 - 4\cos n\pi}{(n\pi)^2} + \frac{4\sin(n\pi)}{(n\pi)^3}$$

$$\Rightarrow a_n = \frac{-2\cos n\pi - 2}{(n\pi)^2} + \frac{4\times 0}{(n\pi)^3} \qquad [\because \sin n\pi = 0]$$

$$\Rightarrow a_n = \frac{-2[\cos n\pi + 1]}{(n\pi)^2}$$

$$\boxed{a_n = \frac{-2[(-1)^n + 1]}{(n\pi)^2}} \text{ --- (III)} \qquad [\because \cos n\pi = (-1)^n]$$

from eqn - (II) & (III) Put the value in eqn - (I)

$$\Rightarrow f(x) = \frac{1}{2\times 3} + \sum_{n=1}^{\infty} \frac{(-2[(-1)^n+1]}{(n\pi)^2}\cos n\pi x$$

$$\text{or} \quad \boxed{f(x) = \frac{1}{6} - \frac{2}{\pi^2}\sum_{n=1}^{\infty}\frac{[(-1)^n+1]}{n^2}\cos(n\pi x)} \text{ Ans}$$