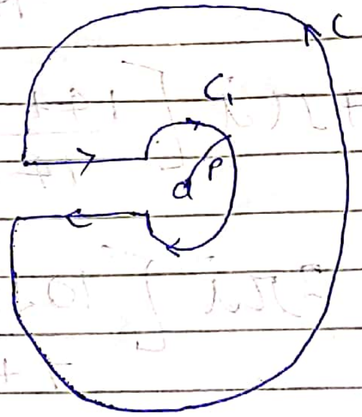
Section-1

(1). (A) **Statement** - If $f(z)$ is analytic with in and on a closed curve c and a is any point with in c then

$$f(a) = \frac{1}{2\pi i} \oint_C \frac{f(z)}{z-a} dz$$

(B) **Proof**: consider the function $f(z)$, which is analytic at every point with in c except at $z=a$. Draw a circle C_1 with a as center and radius ρ such that C_1 lies entirely inside C. Thus $\frac{f(z)}{z-a}$ is analytic in the region between c and C_1



By cauchy theorem we have

$$\oint_C \frac{f(z)}{z-a} dz = \oint_{C_1} \frac{f(z)}{z-a} dz \quad \text{(1)}$$

Now eqn of the circle C_1 is $|z-a| = \rho$ or $z-a = \rho e^{i\theta}$

So that $dz = i\rho e^{i\theta} d\theta$

$$\therefore \oint_{C_1} \frac{f(z)}{z-a}\,dz = \int_0^{2\pi} \frac{f(a+\rho e^{i\theta})}{\rho e^{i\theta}}\, i\rho e^{i\theta}\, d\theta = i\int f(a+\rho e^{i\theta})\,d\theta.$$

Hence by eq. (1) we have

$$\therefore \oint_{C_1} \frac{f(z)}{z-a}\,dz = i\int_0^{2\pi} f(a+\rho e^{i\theta})\,d\theta \quad \text{—(2)}$$

In the limiting form as the circle C_1 shrinks to the point a, i.e. $\rho \to 0$ then the form eqn (2)

$$\oint_C \frac{f(z)}{z-a}\,dz = i\int_0^{2\pi} f(a)\,d\theta = if(a)\int_0^{2\pi} d\theta = 2\pi i f(a)$$

Hence $f(a) = \dfrac{1}{2\pi i}\oint_C \dfrac{f(z)}{z-a}\,dz$

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