Given $f(x) = x - x^2$ in $0 < x < 1$

$\Rightarrow f(x) = x - x^2 \quad (0, 1)$

We know that

Half range cosine series for the function interval $(0, 1)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{l}\right) \quad \text{— (I)}$$

where $a_0 = \frac{2}{L}\int_0^1 f(x)\,dx$

$\Rightarrow a_0 = \frac{2}{1}\int_0^1 (x - x^2)\,dx = 2\int_0^1 (x - x^2)\,dx$

$a_0 = \frac{2}{1}\int_0^1 (x - x^2)\,dx = 2\int_0^1 (x - x^2)\,dx$

$a_0 = 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$

$a_0 = 2\left[\frac{1^2 - 0^2}{2} - \frac{1^3 - 0^3}{3}\right] = 2\left[\frac{1}{2} - \frac{1}{3}\right]$

$a_0 = 2 \times \left(\frac{3-2}{6}\right) = \frac{1}{3}$

$$\boxed{a_0 = \frac{1}{3}} \quad \text{— (II)}$$

$a_n = \frac{2}{L}\int_0^1 f(x) \cos\left(\frac{n\pi x}{L}\right) dx$

$$a_n = \frac{2}{L}\int_0^1 (x - x^2)\cos(n\pi x)\,dx$$

$$a_n = 2\int_0^1 x\cos n\pi x\,dx - 2\int_0^1 x^2\cos n\pi x\,dx$$

$$\Rightarrow a_n = 2\left\{\left[\frac{x\sin(n\pi x)}{(n\pi)}\right]_0^1 + \left[\frac{\cos n\pi x}{(n\pi)^2}\right]_0^1\right\} - 2$$

$$\left\{\left[\frac{x^2 \sin n\pi x}{n\pi}\right]_0^1 - \int_0^1 \frac{2x\sin n\pi x}{(n\pi)}\right) dx$$

$$\Rightarrow a_n = 2\left\{\frac{1\sin n\pi - 0x\sin 0}{n\pi} + \frac{\cos n\pi - \cos 0}{(n\pi)^2}\right\} -$$

$$-2\left\{\frac{1^2 \times \sin n\pi - 0\sin 0}{n\pi} - \frac{2}{n\pi}\int_0^1 x\sin(n\pi x)\,dx\right\}$$

$$\Rightarrow a_n = 2\left\{\frac{\sin n\pi}{n\pi} + \frac{\cos n\pi - 1}{(n\pi)^2}\right\} - 2\left\{\frac{\sin n\pi}{n\pi} - \frac{2}{n\pi}\int_0^1 x\sin n\pi x\,dx\right\}$$

$$\Rightarrow a_n = \frac{2\sin n\pi}{n\pi} + \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \frac{2\sin n\pi}{n\pi} + \frac{4}{n\pi}\left\{ - \left[\frac{x\cos n\pi x}{n\pi}\right]_0^1 + \left[\frac{\sin n\pi x}{n\pi^2}\right]_0^1\right\}$$

$$\Rightarrow a_n = \frac{2\sin \cancel{n\pi}}{\cancel{n\pi}} + \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \frac{2\sin \cancel{n\pi}}{\cancel{n\pi}} + \frac{4}{n\pi}$$

$$\left\{-\left(\frac{\cos n\pi - (\cos 0)x_0}{n\pi}\right) + \frac{\sin n\pi - \sin 0}{(n\pi)^2}\right]$$

$$\Rightarrow a_n = \frac{2(\cos n\pi - 1)}{(n\pi)^2} - \frac{4\cos n\pi}{n\pi^2} + \frac{4}{n\pi}\frac{\sin n\pi}{(n\pi)^2}$$

$$\Rightarrow a_n = \frac{2\cos n\pi - 2}{(n\pi)^2} - \frac{4\cos n\pi}{(n\pi)^2} + \frac{4}{n\pi}\frac{\sin n\pi}{(n\pi)^2}$$

$$\Rightarrow a_n = \frac{2\cos n\pi - 2 - 4\cos n\pi}{(n\pi)^2} + \frac{4\sin(n\pi)}{(n\pi)^3}$$

$$\Rightarrow a_n = \frac{-2\cos n\pi - 2}{(n\pi)^2} + \frac{4\times 0}{(n\pi)^3} \qquad \left[\because \sin n\pi = 0\right]$$

$$\Rightarrow a_n = -2\left[\cos n\pi + 1\right]$$

$$\Rightarrow a_n = \frac{-2\left[(-1)^n + 1\right]}{(n\pi)^2} \quad \text{--- (III)} \qquad \left[\because \cos n\pi = (-1)^n\right]$$

From eqn (II) and (III) put the value in eqn (I)

$$f(x) = \frac{1}{2\times 3} + \sum_{n=1}^{\infty} \frac{(-2)\left[(-1)^n + 1\right]}{(n\pi)^2} \cos n\pi x$$

$$\boxed{f(x) = \frac{1}{6} - \frac{2}{\pi^2} \sum_{n=1}^{\infty} \frac{\left[(-1)^3 + 1\right]}{n^2} \cos(n\pi x)}$$