Time ~~Ind~~ dependent wave eqn.

From Schrodinger's time independent wave eqn.

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E - V)\psi = 0 \quad \text{--- (1)}$$

$$\nabla^2\psi + \frac{2m}{\hbar^2}(E\psi - V\psi) = 0 \quad \text{--- (2)}$$

From general solution.

$$\psi = \psi_0 e^{-i\omega t} \quad \text{--- (3)}$$

diff above eqn w.r.to [t]

$$\frac{\partial\psi}{\partial t} = \psi_0(-i\omega)e^{-i\omega t}$$

$$\frac{\partial\psi}{\partial t} = \psi(-i\omega)$$

where $\omega = 2\pi\nu$ $\left\{ E = h\nu,\ \nu = \frac{E}{h} \right\}$

$$\omega = \frac{2\pi E}{h}$$

$$\omega = \frac{E}{\hbar} \quad \left\{ \because \hbar = \frac{h}{2\pi} \right\}$$

So $\frac{\partial\psi}{\partial t} = \psi(-i)\frac{E}{\hbar}$

Here

$$\cancel{E\psi(-i) = \frac{\partial\psi}{\partial t}}$$

$$E\psi = \frac{\partial\psi}{\partial t}\frac{\hbar}{-i}$$

Date / /
Page No.

$$\boxed{\text{So} \quad E\psi = \hbar i \frac{\partial \psi}{\partial t}}$$ Put this value in eqn ②

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left(E\psi - V\psi\right) = 0 \quad \text{②}$$

$$\nabla^2 \psi + \frac{2m}{\hbar^2}\left(\hbar i \frac{\partial \psi}{\partial t} - V\psi\right) = 0$$

$$\boxed{\frac{\hbar^2}{2m}\nabla^2 \psi + \hbar i \frac{\partial \psi}{\partial t} - V\psi = 0} \quad \text{③}$$

$$\hbar i \frac{\partial \psi}{\partial t} = V\psi - \frac{\hbar^2}{2m}\nabla^2 \psi$$

$$\hbar i \frac{\partial \psi}{\partial t} = \left(V - \frac{\hbar^2}{2m}\nabla^2\right)\psi$$

$$\boxed{E\psi = H\psi}$$

where $H = \left(V - \frac{h^2}{2m}\nabla^2\right)$

is called Hamiltonian operator

$$E\psi = \hbar i \frac{\partial \psi}{\partial t}$$