Que :-1 $f(x) = \begin{cases} -1 & -\pi < x < -\pi/2 \\ 0 & -\pi/2 < x < \pi/2 \\ 1 & \pi/2 < x < \pi \end{cases}$

$b - a = 2\pi$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \quad \text{--- (1)}$$

$$a_0 = \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2} -(1)\,dx + \int_{\pi/2}^{\pi} (1)\,dx\right]$$

$$a_0 = \frac{1}{\pi}\left[-|x|_{-\pi}^{-\pi/2} + |x|_{\pi/2}^{\pi}\right]$$

$$a_0 = \frac{1}{\pi}\left[\cancel{\pi}/2 - \cancel{\pi} + \cancel{\pi} - \cancel{\pi}/2\right]$$

$$a_0 = \frac{1}{\pi}[0] = 0$$

$$\boxed{a_0 = 0} \quad \text{———— (2)}$$

$$a_n = \frac{1}{\pi}\left[\int_{-\pi}^{-\pi/2} -\cos(nx)\,dx + \int_{\pi/2}^{\pi} \cos(nx)\,dx\right]$$

$$a_n = \frac{1}{\pi}\left\{\left[-\frac{\sin nx}{n}\right]_{-\pi/2}^{-\pi/2} + \left[\frac{\sin(nx)}{n}\right]_{\pi/2}^{\pi}\right\}$$

~~We know that $\sin(n\pi/2) = 0$~~

~~So~~

$\sin\left(\frac{3\pi}{2}+\pi/2\right) \quad \pi/2 = \frac{2}{n\pi} \quad \frac{n\pi^2}{4}$

~~Rest~~ Put all values in equ ②.

$$f(x) = 0 + 0 + \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin nx$$

$$f(x) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin nx \quad \text{—⑤.}$$

Put $x = \pi/2$ in equ ⑤ we get,

$$\left[f(x)\right]_{x=\pi/2} = \sum \frac{2}{n\pi}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin n\pi/2$$

$$\frac{0+1}{2} = \frac{2}{\pi}\sum \frac{1}{n}\left(\cos\frac{n\pi}{2} - \cos n\pi\right)\sin n\pi/2$$

putting $n = 1, 2, 3, 4, 5$ ———

$$\pi/4 = \left(\cos\pi/2 - \cos\pi\right)\sin\pi/2 + 0$$

$$= \frac{1}{3}\left(\cos 3\pi/2 - \cos 3\pi\right)\sin 3\pi/2 + 0$$

$$= \frac{1}{5}\left(\cos 5\pi/2 - \cos 5\pi\right)\sin 5\pi/2 + 0 + \text{———}$$

$$\pi/4 = 1 + \frac{1}{3}(-1) + \frac{1}{5}(+1) + \frac{1}{7}(-1) + \text{———}$$

$$\pi/4 = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \text{———}$$