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$$F(x) = x - x^2, \quad 0 < x < 1$$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{1}$$

$$a_0 = 2\int_0^1 f(x)\,dx$$

$$= 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{2}{6} = \frac{1}{3}$$

$$a_n = \frac{2}{1}\int_0^1 f(x)\cos\frac{n\pi x}{1}\,dx$$

$$= 2\int_0^1 (x - x^2)\cos n\pi x\,dx$$

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$$\Rightarrow 2\left[(x-x^2)\left[\frac{\sin n\pi x}{n\pi}\right]-(1-2x)\left[\frac{-\cos n\pi x}{n^2\pi^2}\right]\right.$$

$$\left.+(-2)\left(-\frac{\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= 2\left[(-1)\frac{\cos n\pi}{n^2\pi^2}-\frac{1}{n^2\pi^2}\right]$$

$$= 2\left[\frac{(-1)^{n+1}}{n^2\pi^2}-\frac{1}{n^2\pi^2}\right]$$

$$f(x) = \frac{1}{6}+\sum_{n=1}^{\infty}\frac{2}{n^2\pi^2}\left[(-1)^{n+1}-1\right]$$

$\cos n\pi x$