

Ans Schrodinger's equation, which is the fundamental equation of quantum mechanics, is a wave equation is the variable ψ .

- Time independent Schrodinger wave eq.

Consider a system of stationary wave to be associated with particle and the position coordinate of the particle (x, y, z) and ψ is the periodic displacement at any instant time 't'.

The general wave equation in 3-D in differential form is given as.

$$\nabla^2 \psi = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Where, v = Velocity of wave, and

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \text{Laplacian operator}$$

The wave function may be written as

$$\psi = \psi_0 e^{-i\omega t}$$

Differentiate eq w.r.t time, we get

$$\frac{\partial \psi}{\partial t} = -i \omega \psi_0 e^{-i\omega t}$$

Again differentiating eq, we get

$$\frac{\partial^2 \psi}{\partial t^2} = i^2 \omega^2 \psi_0 e^{-i\omega t}$$

$$\frac{\partial^2 \psi}{\partial t^2} = -\omega^2 \psi$$

Putting these values in eq

$$\nabla^2 \psi = -\frac{\omega^2}{v^2} \psi$$

But

$$\omega = 2\pi v = \frac{2\pi v}{\lambda} = \frac{\omega}{v} = \frac{2\pi}{\lambda}$$

From eq and eq, we get

$$\nabla^2 \psi = -\frac{4\pi^2}{\lambda^2} \psi$$

From de - Broglie's wavelength

$$\lambda = \frac{h}{mv}$$

then,

$$\nabla^2 \psi = \frac{-4\pi^2 m^2 v^2}{h^2} \psi$$

if E and V are the total energy and potential energy of a particle of E_k is kinetic energy, then

$$E_k = E - V \text{ or } \frac{1}{2} m v^2 = E - V$$

$$m^2 v^2 = 2m(E - V)$$

From (1) and (2) we get

$$\nabla^2 \psi = \frac{-4\pi^2 2m [E - V]}{h^2} \psi$$

$$\nabla^2 \psi + \frac{2m [E - V]}{h^2} \psi = 0$$

This is required time-independent Schrödinger wave equation