- The mass - energy equation ($E = mc^2$) is the most famous and most significant relationship obtained by einstein from the postulates of their special theory of relativity.

- Numerous experiments have demostrated that the mass is convertible into energy, and energy into mass,

- Let us consider a particle of mass s acted upon by a force F in the same direction as its velocity v.

- Increase in energy of the particle by the application of force may be defined in terms of work which is the product of force and displacement (due to application of force)

- Thus if a force F displaces the particle through a small distance ds, then work done,

dW is stored by the particle as its kinetic energy dK, therefore

$$dW = dK = F \cdot ds \quad \text{—— (1)}$$

According to Newton law of motion in relativistic mechanics.

the force is the rate of change of momentum P, that is

$$F = \frac{dP}{dt} = \frac{d}{dt}(mv) \quad \text{—} \textcircled{2}$$

According to the theory of relativity,

mass of the particle varies with velocity. Hence m and v both are variable in equation (2). therefore.

$$F = m\frac{dv}{dt} + v\frac{dm}{dt} \quad \text{—} \textcircled{3}$$

Substituting this value of F from equation (3) in equation (1), we get

$$dK = m\frac{dv}{dt}ds + v\frac{dm}{dt}ds$$

$$= m\frac{ds}{dt}dv + v\frac{ds}{dt}dm$$

or

$$dk = mv\,dv + v^2\,dm \qquad (\because ds/dt = v) \quad \text{—— (4)}$$

According to einstein special theory of relativity, the mass m of a particle moving with velocity V varies in accordance with the relation.

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}} = m_0\left[1-\frac{v^2}{c^2}\right]^{-1/2} \quad \text{—— (5)}$$

where m_0 is the rest mass of the particle

Differentiating equation (5), we get

$$dm = m_0\left(\frac{-1}{2}\right)\left(1-\frac{v^2}{c^2}\right)^{-3/2}\left(-\frac{2v\,dv}{c^2}\right) = \frac{m_0 v\,dv}{c^2[1-(v^2/c^2)]^{3/2}}$$

But, from equation (5)

$$m_0 = m(1-v^2/c^2)^{1/2}$$

$$dm = \frac{m(1-v^2/c^2)^{1/2}\,v\,dv}{c^2\left(1-\frac{v^2}{c^2}\right)^{3/2}}$$

or

$$dm = \frac{mv\,dv}{(c^2-v^2)}$$

or $\quad mv\,dv = (c^2-v^2)\,dm$

Substituting this Value of $mv\,dv$ in eq (4), we get

$$dK = (c^2-v^2)\,dm + v^2 dm = c^2 dm$$

if the Changes in kinetic energy of the particle be K, when it mass changes from rest mass m_0 to effective mass m, then

$$K = \int dK = \int_{m_0}^{m} c^2 dm = c^2(m-m_0)$$

$$K = c^2(m-m_0) = c^2\left[\frac{m_0}{\sqrt{1-v^2/c^2}} - m_0\right]$$

This is the relativistic expression for kinetic energy of a particle.

From this expression it is clear that the increase in kinetic energy is due to the increase in mass of the particle on account of its relative motion and is equal to the product of gain in mass and square of the velocity of light

Therefore, m_0c^2 may be regarded as the rest energy of the particle of rest mass m_0.

This rest energy may be considered as internal stored energy of the particle.

The total energy of a moving particle is the sum of kinetic energy of motion and its energy at rest, that is,

Total energy

$$E = \text{rest energy} + \text{relativistic K.E.}$$

$$= m_0c^2 + (m - m_0)c^2 = mc^2$$

$$E = mc^2$$

This is well known Einstein mass-energy relation which states a universal equivalence b/w mass and energy