$f(z) = u + iv = \log z = \log(x+iy)$

Let $x = r\cos\theta$ & $y = r\sin\theta$ so that

$x + iy = r(\cos\theta + i\sin\theta) = re^{i\theta}$

$\log(x+iy) = \log(re^{i\theta}) = \log r + i\theta$

$= \frac{1}{2}\log(x^2+y^2) + i\tan^{-1}(y/x)$

Seperating the ~~[illegible]~~ real & img. part,

$u = \frac{1}{2}\log(x^2+y^2)$ & $v = \tan^{-1}(y/x)$

$$\frac{\partial u}{\partial x} = \frac{x}{x^2+y^2}, \quad \frac{\partial u}{\partial y} = \frac{y}{x^2+y^2}$$

$$\frac{\partial u}{\partial x} = \frac{-y}{y^2+x^2}, \quad \frac{\partial v}{\partial y} = \frac{x}{x^2+y^2}$$

We observe the C-R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \& \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

are satisfied when $x^2+y^2 = 0$ i.e

$x = 0, y = 0$.

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