Q. $\iiint_D dx\,dy\,dz$.

$$\frac{x^2}{a^2} = u, \quad \frac{y^2}{b^2} = v, \quad \frac{z^2}{c^2} = w$$

$$2x\,dx = a^2\,du$$

$$dx = \frac{a\,du}{2\sqrt{u}}$$

Similarly $\quad dy = \frac{b\,dv}{2\sqrt{v}}$

$$dz = \frac{c\,dw}{2\sqrt{w}}$$

Required volume

$$V = \iiint_{V} \frac{abc}{8\sqrt{uvw}}\,du\,dv\,dw$$

where D' → Region when $u \geq 0, v \geq 0$

$w \geq 0$ &

$u + v + w = 1$

$$= \frac{abc}{8} \iiint_{D'} u^{-1/2} v^{-1/2} w^{-1/2}\,du\,dv\,dw$$

using dirichlet integral,

$$\frac{abc}{8} \frac{\lfloor\frac{1}{2} \lfloor\frac{1}{2} \lfloor\frac{1}{2}}{\lfloor\frac{1}{2} + \frac{1}{2} + \frac{1}{2} + 1} = \frac{abc}{8} \frac{(\sqrt{\pi})^3}{\frac{3}{2}\frac{1}{2}\sqrt{\pi}}$$

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$= \frac{\pi}{8} abc$ cubic unit

Mass = Volume × density

$= \iiint_D kxyz \, dx\,dy\,dz$

$x = au^2$ and $dx = \frac{a}{2\sqrt{u}} du$

Similarly $y = bv^2$ & $dy = \frac{b}{2\sqrt{v}} dv$

$z = cw^2$ and $dz = \frac{c}{2\sqrt{w}} dw$

$$\text{Mass} = \iiint_{D'} \frac{abc}{8} u^{-1/2} v^{-1/2} w^{-1/2} \; k\, a\sqrt{u}\, b\sqrt{v}\, c\sqrt{w} \; du\,dv\,dw$$

$$= \frac{Ka^2b^2c^2}{8} \iiint_{D'} u^0 v^0 w^0 \, du\,dv\,dw$$

$$\frac{Ka^2b^2c^2}{8} \iiint_{D'} u^{1-1} v^{1-1} w^{1-1} \, du\,dv\,dw$$

$$= \frac{Ka^2b^2c^2}{8} \frac{\Gamma 1 \, \Gamma 1 \, \Gamma 1}{\Gamma 1+1+1+1} = \frac{Ka^2b^2c^2}{48}$$

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