$$\frac{d^2y}{dx^2} + (3\sin x - \cot x)\frac{dy}{dx} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$

→ on comparing given equations with

$$y'' + Py' + Qy = R$$

$P = 3\sin - \cot x \qquad Q = 2\sin^2 x$

$R = e^{-\cos x}\sin^2 x.$

Choose z such that $\left(\frac{dz}{dx}\right)^2$

$= \sin^2 x \Rightarrow \frac{dz}{dx} = \sin x$

$z = -\cos x$

$\frac{d^2z}{dx^2} = \cos x$

$P_1 = \frac{\cos x + (3\sin x - \cot x)\sin x}{\sin^2 x}$

$Q_1 = \frac{2\sin^2 x}{\sin^2 x}, = 2$

$R_1 = \frac{e^{-\cos x}\sin^2 x}{\sin^2 x} = e^{-\cos x} = e^{x}$

$\Rightarrow \frac{d^2y}{dz^2} + \frac{3dy}{dz} + 2y = e^z.$

A.E; → $m^2 + 3m + 2 = 0$

$m = -1, -2$

$$C.F = c_1 e^{-2} + c_2 e^{-2x}$$

$$P.I = \frac{1}{D^2+3D+2}(e^x) = \frac{e^x}{6}$$

Complete solution :-

$$y = c_1 e^{-2} + c_2 e^{-2x} + \frac{e^x}{6}$$

$$y = c_1 e^{\cos x} + c_2 e^{2\cos x} + \frac{e^{-\cos x}}{6}$$