Section - 1

Ques 2:- $f(x) = x - x^2$ in $0 < x < 1$

(Half Range series)

$f(x) = x - x^2, \quad 0 < x < 1$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{1}$$

$$a_0 = 2\int_0^1 f(x)\,dx = 2\int_0^1 (x - x^2)\,dx$$

$$= 2\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = 2\left[\frac{1}{2} - \frac{1}{3}\right] = \frac{2}{6} = 1/3.$$

$$a_n = \frac{2}{1.}\int_0^1 f(x) \cos \frac{n\pi x}{1}\,dx$$

$$= 2\int_0^1 (x - x^2) \cos n\pi x\,dx$$

$$= \left[(x - x^2)\left(\frac{\sin n\pi x}{n\pi}\right) - (1 - 2x)\left(\frac{-\cos n\pi x}{n^2\pi^2}\right) + (-2)\left(\frac{-\sin n\pi x}{n^3\pi^3}\right)\right]_0^1$$

$$= 2\left[\frac{(-1)\cos n\cdot\pi}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right] = 2\left[\frac{(-1)^{n+1}}{n^2\pi^2} - \frac{1}{n^2\pi^2}\right]$$

$$f(x) = \frac{1}{6} + \sum_{n=1}^{\infty} \frac{2}{n^2\pi^2}\left((-1)^{n+1} - 1\right)\cos n\pi x$$