Q. $f(z) = u + iv$ — (1)

$|f(z)| = \sqrt{u^2 + v^2}$ — (2)

Partially differentiating eq^n w.r.t x and y,

$$\frac{\partial}{\partial x}|f(z)| = \frac{1}{2}(u^2+v^2)^{-1/2}\left(2u\frac{\partial u}{\partial x} + 2v\frac{\partial v}{\partial x}\right)$$

$$= \frac{u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}}{|f(z)|} \quad — (3)$$

$$\text{i.y} \quad \frac{\partial}{\partial y}|f(z)| = \frac{u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y}}{|f(z)|} \quad — (4)$$

Squaring & adding 3 & 4.

$$\left\{\frac{\partial}{\partial x}|f(z)|\right\}^2 + \left\{\frac{\partial}{\partial y}|f(z)|\right\}^2 = \frac{\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right)^2 + \left(u\frac{\partial u}{\partial y} + v\frac{\partial v}{\partial y}\right)^2}{|f(z)|^2}$$

$$= \frac{\left(u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right)^2 + \left(-u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial x}\right)^2}{|f(z)|^2}$$

using C-R eqns

$$\frac{(u^2+v^2)\left[\left(\frac{\partial u}{\partial x}\right)^2+\left(\frac{\partial v}{\partial x}\right)^2\right]}{|f'(z)|^2}$$

$$= \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2 \quad \left(\because |f(z)|^2 = u^2+v^2\right)$$

$$= |f'(z)|^2 \quad \left(\because f'(z) = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right)$$