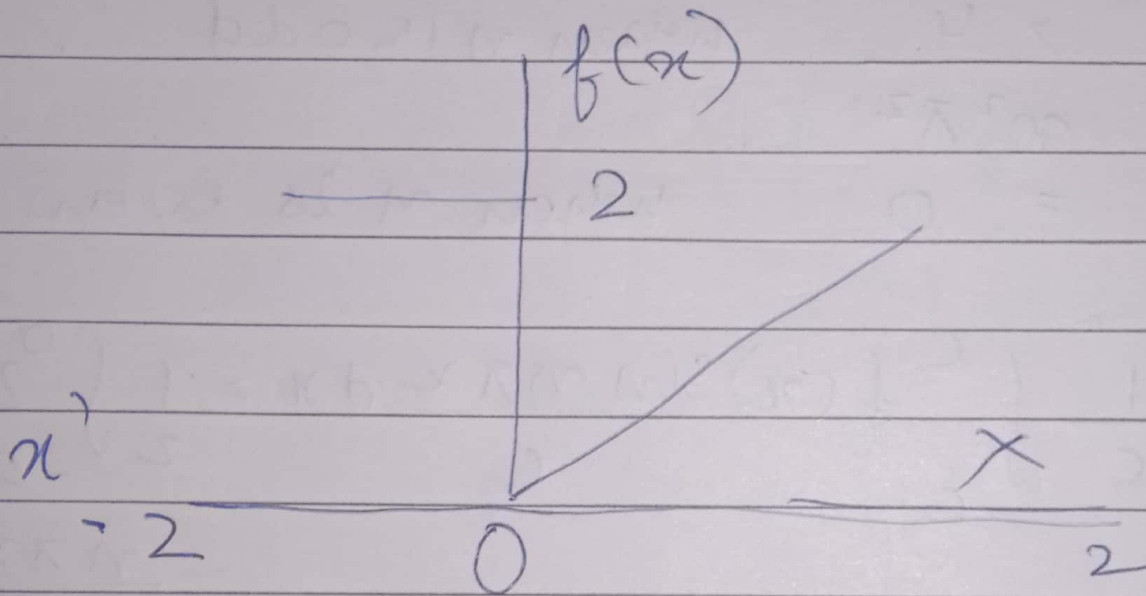


$$f(x) = \begin{cases} 2 & \text{in } -2 \leq x \leq 0 \\ x & \text{in } 0 < x < 2. \end{cases}$$



Interval is $(-2, 2)$ & $c = 2$.

$$a_0 = \frac{1}{c} \int_{-c}^c f(x) dx = \frac{1}{2} \left[\int_{-2}^0 2 dx + \int_{-2}^0 x dx \right]$$

$$= \frac{1}{2} \left[[2x]_{-2}^0 + \left(\frac{x^2}{2} \right)_{-2}^0 \right] = \frac{1}{2} [4 + 2] = 3$$

$$a_n = \frac{1}{c} \int_{-c}^c f(x) \cos\left(\frac{n\pi x}{c}\right) dx = \frac{1}{2} \left[\int_{-2}^0 2 \cos\left(\frac{n\pi x}{2}\right) dx + \int_0^2 x \cos\left(\frac{n\pi x}{2}\right) dx \right]$$

$$= \frac{1}{2} \left[\frac{4}{n\pi} \left(\frac{\sin n\pi x}{2} \right) \Big|_{-2}^0 + \left(\frac{x^2 \sin n\pi x}{2n\pi} + \frac{4}{n^2\pi^2} \cos n\pi x \right) \Big|_0^2 \right]$$

$$= \frac{1}{2} \left[\frac{4}{n^2\pi^2} \cos n\pi - \frac{4}{n^2\pi^2} \right] = \frac{2}{n^2\pi^2} [(-1)^n - 1]$$

$$= \frac{4}{n^2\pi^2} \quad \text{when } n \text{ is odd}$$

$$= 0 \quad \text{when } n \text{ is even}$$

$$b_n = \frac{1}{c} \int_{-c}^c f(x) \sin\left(\frac{n\pi x}{c}\right) dx = \frac{1}{2} \left[\int_{-2}^0 2 \sin\left(\frac{n\pi x}{2}\right) dx + \int_0^2 x \sin\left(\frac{n\pi x}{2}\right) dx \right]$$

$$= \frac{1}{2} \left[2 \left(\frac{-2 \cos n\pi x}{n\pi} \right) \Big|_{-2}^0 \right] + \frac{1}{2} \left[x \left(\frac{-2 \cos n\pi x}{n\pi} \right) \Big|_0^2 \right]$$

$$+ \left(\frac{4 \sin n\pi x}{n^2\pi^2} \right) \Big|_0^2$$

$$= \frac{1}{2} \left[\frac{-4}{n\pi} + \frac{4}{n\pi} \cos n\pi \right] + \frac{1}{2} \left[\frac{-4}{n\pi} \cos n\pi + \frac{4}{n^2\pi^2} \sin n\pi \right]$$

$$= \frac{1}{2} \left[-\frac{4}{n\pi} \right] = -\frac{2}{n\pi}$$

$$f(x) = \frac{a_0}{2} + \frac{a_1 \cos \pi x}{c} + \frac{a_2 \cos 2\pi x}{c} + \frac{a_3 \cos 3\pi x}{c} +$$

...

$$+ \frac{b_1 \sin \pi x}{c} + \frac{b_2 \sin 2\pi x}{c} +$$

$$\frac{b_3 \sin 3\pi x}{c} + \dots$$

$$= \frac{3}{2} - \frac{4}{\pi^2} \left[\frac{1}{1^2} \frac{\cos \pi x}{2} + \frac{1}{3^2} \frac{\cos 3\pi x}{2} + \dots \right]$$

$$- \frac{2}{\pi} \left[\frac{\sin \pi x}{2} + \frac{\sin 2\pi x}{2} + \dots \right]$$

$$+ \frac{1}{2} \frac{\sin 3\pi x}{2} + \dots \Bigg\}$$