Dirichlet's Integral for Three Variables

$$\iiint x^{l-1} y^{m-1} z^{n-1}\, dx\, dy\, dz$$

$$= \frac{\Gamma l\, \Gamma m\, \Gamma n}{\Gamma(l+m+n+1)}$$

D is the domain

$x \geq 0,\ y \geq 0,\ z \geq 0$ &

$x+y+z \leq 1$

Let, $x+y+z \leq 1$

$y+z \leq 1-x = a$ (Let)

$$I = \int_0^1 \int_0^{1-x} \int_0^{1-x-y} x^{l-1} y^{m-1} z^{n-1}\, dz\, dy\, dx$$

$$= \int_0^1 x^{l-1} \left[\int_0^a \int_0^{a-y} y^{m-1} z^{n-1}\, dz\, dy \right] dx$$

$$= \int_0^1 x^{l-1} \frac{\Gamma m\, \Gamma n}{\Gamma(m+n+1)}\, a^{m+n}\, dx$$

$$\frac{\Gamma m\, \Gamma n}{\Gamma(m+n+1)} \int_0^1 x^{l-1} (1-x)^{m+n+1-1}\, dx$$

$$\frac{\Gamma m\, \Gamma n}{\Gamma(m+n+1)}\, B(l, m+n+1)$$

$$\frac{\Gamma m\, \Gamma n}{\Gamma(m+n+1)} \cdot \frac{\Gamma l\, \Gamma(m+n+1)}{\Gamma(l+m+n+1)} = \frac{\Gamma l\, \Gamma m\, \Gamma n}{\Gamma(l+m+n+1)}$$