

Section-6

Ans 1 Let y be any arbitrary element of $f(A \cup B)$, then

$$y \in f(A \cup B) \Rightarrow y = f(x), \text{ where } x \in A \cup B$$

$$\Rightarrow y = f(x), \text{ where } x \in A \text{ or } x \in B$$

$$\Rightarrow y = f(x) \in f(A) \text{ or } y = f(x) \in f(B)$$

$$\Rightarrow y = f(x) \in [f(A) \cup f(B)]$$

$$= y \in f(A) \cup f(B)$$

$$f(A \cup B) \subseteq f(A) \cup f(B).$$

Again, let y be any arbitrary element of $f(A) \cup f(B)$, then

$$y \in f(A) \cup f(B) \Rightarrow y \in f(A) \text{ or } y \in f(B)$$

$$\Rightarrow y = f(x), \text{ where } x \in A \text{ or } x \in B$$

$$\Rightarrow y = f(x), \text{ where } x \in A \cup B$$

$$\Rightarrow y = f(x), \text{ where } f(x) \in f(A \cup B)$$

$$\Rightarrow y \in f(A \cup B)$$

$$f(A) \cup f(B) \subseteq f(A \cup B)$$

Hence, from (1) & (2), we get $f(A \cup B) = f(A) \cup f(B)$ Ans