

SECTION-3

ANS-1

Answer (i)

Clearly out of the 370 students who read newspaper, 23 must read both as simply adding given 393 students,

Proof by set theory

$$A \cup B = A + B - A \cap B$$

$$A \cap B = 200 + 193 - 370$$

$$A \cap B = 23.$$

23 students reads both the news paper

The equation of the given planes are -

$$my + nz = 0 \quad \text{--- (1)}$$

$$nz + lx = 0 \quad \text{--- (2)}$$

$$lx + my = 0 \quad \text{--- (3)}$$

$$lx + my + nz = p \quad \text{--- (4)}$$

Solving (1), (2) and (3) we get $x=0, y=0, z=0$

Solving (2), (3) and (4) we get $x = \frac{p}{l}, y = \frac{p}{m}, z = \frac{p}{n}$

Solving (1), (3) and (4) we get $x = \frac{p}{l}, y = -\frac{p}{m}, z = \frac{p}{n}$

Solving (1), (2) and (4) we get $x = \frac{p}{l}, y = \frac{p}{m}, z = -\frac{p}{n}$

Hence, the coordinates of the vertices of the tetrahedron are -

$$(0, 0, 0) : \left(-\frac{p}{l}, \frac{p}{m}, \frac{p}{n}\right) : \left(\frac{p}{l}, -\frac{p}{m}, -\frac{p}{n}\right) \text{ and } \left(\frac{p}{l}, \frac{p}{m}, -\frac{p}{n}\right)$$

Therefore, the volume v of the tetrahedron -

$$= \frac{1}{6} \begin{vmatrix} -\frac{p}{l} & \frac{p}{m} & \frac{p}{n} \\ \frac{p}{l} & -\frac{p}{m} & \frac{p}{n} \\ \frac{p}{l} & \frac{p}{m} & -\frac{p}{n} \\ 1 & 1 & 1 \end{vmatrix}$$

Answer (ii)

We know that -

$$-1 < \cos 3x < 1$$

multiply by -1 on all the sides we get,

$$1 > -\cos 3x > -1 \quad [\text{as multiplying by } -1 \text{ changes the inequality sign}]$$

add 2 on all the sides we get,

$$3 > 2 - \cos 3x > 1$$

now take the reciprocal of this whole expression we get -

$$\frac{1}{3} < \frac{1}{(2 - \cos 3x)} < 1$$

$$= \frac{p^3}{6 \text{ mm}} \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \frac{p^3}{6 \text{ mm}} \begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} \begin{cases} c_2 + c_1 \\ c_3 + c_1 \end{cases}$$

$$= \frac{p^3}{6 \text{ mm}} \times 4$$

$$= \frac{2p^3}{3 \text{ mm}} \text{ ~~q.e.d.~~$$

Proved