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De Morgan's Laws -

De Morgan's Law has two important theorems based on boolean algebra. The general form of these theorems are -

$$\overline{(A+B+C+D+\dots+F)} = \bar{A} \cdot \bar{B} \cdot \bar{C} \cdot \bar{D} \cdot \dots \cdot \bar{F}$$

$$\overline{(A \cdot B \cdot C \cdot D \cdot \dots \cdot F)} = \bar{A} + \bar{B} + \bar{C} + \bar{D} + \dots + \bar{F}$$

The generalised form of De Morgan's theorem states that the complement of a function obtained by interchanging AND and OR operator and complementing each literal. It is very important in dealing with NOR and NAND gates.

The theorems with 2 variables are given below -

i) $\overline{A \cdot B} = \bar{A} + \bar{B}$

i.e. The complement of a product is equal to the ~~product~~ sum of complements of the variables.

~~Proof:~~

~~is to prove $\overline{A \cdot B} = \bar{A} + \bar{B}$~~

ii) $\overline{A+B} = \bar{A} \cdot \bar{B}$

i.e. The complement of a sum is equal to the product of complements of the variables.

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Proofs:

is to prove $\overline{A \cdot B} = \overline{A + B}$

To prove that $(\overline{A + B})$ is the complement of $(A \cdot B)$, we need to show that

→ by ORing $(A \cdot B)$ with $(\overline{A + B})$ gives 1
i.e. $(A \cdot B) + (\overline{A + B}) = 1$

and

→ by ANDing $(A \cdot B)$ with $(\overline{A + B})$ gives complemented value 0
i.e. $(A \cdot B) \cdot (\overline{A + B}) = 0$

So, first show that $(A \cdot B) + (\overline{A + B}) = 1$

$$\begin{aligned}
 \text{L.H.S} &= (A \cdot B) + (\overline{A + B}) \\
 &= (\overline{A + B}) + (A \cdot B) \quad \text{[commutative law]} \\
 &= (\overline{A + B} + A) \cdot (\overline{A + B} + B) \quad \text{[Distributive law]} \\
 &= (\overline{A} + A + \overline{B}) \cdot (\overline{A} + B + B) \quad \text{[commutative law]} \\
 &= ((A + \overline{A}) + \overline{B}) \cdot (\overline{A} + (B + B)) \quad \text{[commutative law]} \\
 &= (1 + \overline{B}) \cdot (\overline{A} + 1) \quad \text{[}\because A + \overline{A} = 1 \text{ and } B + \overline{B} = 1\text{]} \\
 &= 1 \cdot 1 \\
 &= 1 = \text{R.H.S} \quad \text{--- (1)}
 \end{aligned}$$

Then show that $(A \cdot B) \cdot (\overline{A + B}) = 0$

$$\begin{aligned}
 \text{L.H.S} &= (A \cdot B) \cdot (\overline{A + B}) \\
 &= ((A \cdot B) + \overline{A}) + ((A \cdot B) \cdot \overline{B}) \quad \text{[Distributive law]} \\
 &= (\overline{A} \cdot (A \cdot B)) + ((A \cdot B) \cdot \overline{B}) \quad \text{[commutative law]} \\
 &= ((\overline{A} \cdot A) \cdot B) + (A \cdot (B \cdot \overline{B})) \quad \text{[Associative law]} \\
 &= ((A \cdot \overline{A}) \cdot B) + (A \cdot (B \cdot \overline{B})) \quad \text{[commutative law]} \\
 &= (0 \cdot B) + (A \cdot 0) \quad \text{[}\because A \cdot \overline{A} = 0 \text{ and } B \cdot \overline{B} = 0\text{]} \\
 &= (B \cdot 0) + (A \cdot 0) \quad \text{[commutative law]}
 \end{aligned}$$

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$$= 0 + 0 \quad [\because B \cdot 0 = 0 \text{ and } A \cdot 0 = 0]$$

$$= 0 = R.H.S \text{ ————— (2)}$$

(1) and (2) show that $\overline{A+B} = \overline{A} \cdot \overline{B}$
 Hence, proved $\overline{A \cdot B} = \overline{A} + \overline{B}$

ii) To prove $\overline{A \cdot B} = \overline{A} + \overline{B}$
 To prove that $(\overline{A \cdot B})$ is the complement of $(A+B)$, we need to show that

→ by OR ing $(A+B)$ with $(\overline{A \cdot B})$ gives 1
 i.e. $(A+B) + (\overline{A \cdot B}) = 1$

and

→ by AND ing $(A+B)$ with $(\overline{A \cdot B})$ gives complemented value 0
 i.e. $(A+B) \cdot (\overline{A \cdot B}) = 0$

So, first show that $(A+B) + (\overline{A \cdot B}) = 1$

$$L.H.S \cdot (A+B) + (\overline{A \cdot B})$$

$$= (A+B + \overline{A}) \cdot (A+B + \overline{B})$$

$$= (A + \overline{A} + B) \cdot (A+B + \overline{B})$$

$$= (1+B) \cdot (A+1)$$

$$= (B+1) \cdot (A+1)$$

$$= 1 \cdot 1$$

$$= 1 = R.H.S. \text{ ————— (1)}$$

Then show that $(A+B) \cdot (\overline{A \cdot B}) = 0$

$$L.H.S = (A+B) \cdot (\overline{A \cdot B})$$

$$= (\overline{A \cdot B}) \cdot (A+B)$$

$$= ((\overline{A \cdot B}) \cdot A) + ((\overline{A \cdot B}) \cdot B)$$

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$$\begin{aligned} &= (\bar{B} \cdot (\bar{A} \cdot A)) + (\bar{A} \cdot (\bar{B} \cdot B)) \\ &= (\bar{B} \cdot (A \cdot \bar{A})) + (\bar{A} \cdot (B \cdot \bar{B})) \\ &= (\bar{B} \cdot 0) + (\bar{A} \cdot 0) \\ &= 0 + 0 \\ &= 0 = R.H.S \quad \text{--- (2)} \end{aligned}$$

① and ② show that $\overline{A \cdot B} = \overline{A+B}$

Hence, proved $\overline{A+B} = \overline{A \cdot B}$

De Morgan Applications-

De Morgan's theorem is useful in the implementation of the basic gate operations with alternative gates, particularly with NAND and NOR gates which are readily available in IC form.