

SECTION - 6

Ans-1

Answer.

Proof -

Let $A, B \subseteq X$ to prove that $f(A \cup B) \subseteq f(A) \cup f(B)$

Let $y \in f(A \cup B)$ this means there exists an element $x \in A \cup B$ with $f(x) = y$. If $x \in A$ we have $y = f(x) \in f(A) \Rightarrow y \in f(A) \cup f(B)$.

If $x \in B$, we have $y = f(x) \in f(B) \Rightarrow y \in f(A) \cup f(B)$

To prove that $f(A) \cup f(B) \subseteq f(A \cup B)$, let

$y \in f(A) \cup f(B)$. If $y \in f(A)$, there exists an element $x \in A$ with $f(x) = y$. Since $x \in A$ we have $x \in A \cup B$, so $y = f(x) \in f(A \cup B)$. If $y \in f(B)$, there exists an element $x \in B$ with $f(x) = y$. Since $x \in B$, we have $x \in A \cup B$, so $y = f(x) \in f(A \cup B)$.

Hence, $f(A \cup B) = f(A) \cup f(B)$

Proved