SECTION - 3

An - 1

ANSWER

The equation of the given planes are -

$my + nz = 0$ ——— ①

$nz + lx = 0$ ——— ②

$lx + my = 0$ ——— ③

$lx + my + nz = p$ ——— ④

Solving ①, ② and ③ we get $x = 0, y = 0, z = 0$

Solving ②, ③ and ④ we get $x = \frac{-p}{l}, y = \frac{p}{m}, z = \frac{p}{n}$

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Hence, the coordinates of the vertices of the tetrahedron are -

$(0,0,0) : \left(\frac{-p}{l}, \frac{p}{m}, \frac{p}{n}\right) : \left(\frac{p}{l}, \frac{-p}{m}, \frac{-p}{n}\right)$ and $\left(\frac{p}{l}, \frac{p}{m}, \frac{-p}{n}\right)$

Therefore, the volume, v of the tetrahedron -

$$= \frac{1}{6}\begin{vmatrix} \frac{-p}{l} & \frac{p}{m} & \frac{p}{n} \\ \frac{p}{l} & \frac{-p}{m} & \frac{p}{n} \\ \frac{p}{1} & \frac{p}{m} & \frac{-p}{n} \end{vmatrix}$$

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$$= \frac{p^3}{6lmn}\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

$$= \frac{p^3}{6lmn}\begin{vmatrix} -1 & 0 & 0 \\ 1 & 0 & 2 \\ 1 & 2 & 0 \end{vmatrix} \quad \begin{Bmatrix} C_2 + C_1 \\ C_3 + C_1 \end{Bmatrix}$$

$$= \frac{p^3}{6lmn} \times 4$$

$$= \frac{2p^3}{3lmn} \quad \text{Ans}$$

Proved