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De Morgan's laws-

De Morgan's law has two important theorems based on boolean algebra the general form of these theorems are-

$$\overline{(A+B+C+D+\dots+F)} = \bar{A}\cdot\bar{B}\cdot\bar{C}\cdot\bar{D}\dots\bar{F}$$

$$\overline{(A\cdot B\cdot C\cdot D\dots F)} = \bar{A}+\bar{B}+\bar{C}+\bar{D}+\dots+\bar{F}$$

The generalised form of De Morgan's theorem stated that the complement of a function obtained by interchanging AND and OR operator and complementing each literal. It is very important in dealing with NOR and NAND gates.

The theorems with 2 variables are given below-

i, $\overline{A\cdot B} = \bar{A}+\bar{B}$

i.e. The complement of a product is equal to the ~~product~~ sum of complements of the variables.

~~Example~~ :

~~i, To prove $\overline{A\cdot B} = \bar{A}+\bar{B}$~~

ii, $\overline{A+B} = \bar{A}\cdot\bar{B}$

i.e. The complement of a product is equal to the product of complements of the variables.

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Proofs:

is to prove $\overline{A \cdot B} = \overline{A} + \overline{B}$

To prove that $(\overline{A} + \overline{B})$ is the complement of $(\overline{A \cdot B})$, we need to show that

→ by ORing $(A \cdot B)$ with $(\overline{A} + \overline{B})$ gives 1

i.e. $(A \cdot B) + (\overline{A} + \overline{B}) = 1$

and

→ by ANDing $(A \cdot B)$ with $(\overline{A} + \overline{B})$ gives complemented value 1

i.e. $(A \cdot B) \cdot (\overline{A} + \overline{B}) = 0$

So, first show that $(A \cdot B) + (\overline{A} + \overline{B}) = 1$

$L.H.S = (AB) + (\overline{A} + \overline{B})$

$= (\overline{A} + \overline{B}) + (A \cdot B)$ (commutative law)

$= (\overline{A} + \overline{B} + A) \cdot (\overline{A} + \overline{B} + B)$ (Distributive law)

$= (\overline{A} + A + \overline{B}) \cdot (\overline{A} + \overline{B} + B)$ (commutative law)

$= ((A + \overline{A}) + \overline{B}) \cdot (\overline{A} + (B + \overline{B}))$ (commutative law)

$= (1 + \overline{B}) \cdot (\overline{A} + 1)$ [$\because A + \overline{A} = 1$ and $B + \overline{B} = 1$]

$= 1 \cdot 1$

$= 1 = R.H.S$ —— (1)

Then show that $(A B) \cdot (\overline{A} + \overline{B}) = 0$

$LH.S = (A \cdot B) \cdot (\overline{A} + \overline{B})$

$= ((A \cdot B) + \overline{A}) + ((A \cdot B) \cdot \overline{B})$ [Distributive law]

$= (\overline{A} \cdot (A \cdot B)) + ((A \cdot B) \cdot \overline{B})$ [Commutative law]

$= ((\overline{A} \cdot A) \cdot B) + (A \cdot (B \cdot \overline{B}))$ [Associative law]

$= ((A \cdot \overline{A}) \cdot B) + (A \cdot (B \cdot \overline{B}))$ [Commutative law]

$= (0 \cdot B) + (A \cdot 0)$ [$\because A \cdot \overline{A} = 0$ and $B \cdot \overline{B} = 0$]

$= (B \cdot 0) + (A \cdot 0)$ [commutative law]

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$= 0+0 \quad [\because B\cdot 0 = 0 \text{ and } A\cdot 0 = 0]$

$= 0 = R.H.S$ ——— (2)

(1) and (2) show that $\bar{A}+\bar{B} = \overline{A\cdot B}$

Hence, proved $\overline{A\cdot B} = \bar{A}+\bar{B}$

ii) To prove $\overline{A+B} = \bar{A}\cdot\bar{B}$

To prove that $(\bar{A}\cdot\bar{B})$ is the complement of $(\overline{A+B})$, we need to show that

→ by ORing $(A+B)$ with $(\bar{A}\cdot\bar{B})$ gives 1

i.e. $(A+B)+(\bar{A}\cdot\bar{B}) = 1$

and

→ by ANDing $(A+B)$ with $(\bar{A}\cdot\bar{B})$ gives complemented value 0

i.e. $(A+B)\cdot(\bar{A}\cdot\bar{B}) = 0$

So, first show that $(A+B)+(\bar{A}\cdot\bar{B}) = 1$

$L.H.S. \quad (A+B)+(\bar{A}\cdot\bar{B})$

$= (A+B+\bar{A})\cdot(A+B+\bar{B})$

$= (A+\bar{A}+B)\cdot(A+B+\bar{B})$

$= (1+B)\cdot(A+1)$

$= (B+1)\cdot(A+1)$

$= 1\cdot 1$

$= 1 = R.H.S.$ ——— (1)

Then show that $(A+B)\cdot(\bar{A}\cdot\bar{B}) = 0$

$L.H.S = (A+B)\cdot(\bar{A}\cdot\bar{B})$

$= (\bar{A}\cdot\bar{B})\cdot(A+B)$

$= ((\bar{A}\cdot\bar{B})\cdot A) + ((\bar{A}\cdot\bar{B})\cdot B)$

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$= (\bar{B} \cdot (\bar{A} \cdot A)) + (\bar{A} \cdot (\bar{B} \cdot B))$

$= (\bar{B} \cdot (A \cdot \bar{A})) + (\bar{A} \cdot (B \cdot \bar{B}))$

$= (\bar{B} \cdot 0) + (\bar{A} \cdot 0)$

$= 0 + 0$

$= 0 = R.H.S$ ——— (2)

(1) and (2) show that $\bar{A} \cdot \bar{B} = \overline{A+B}$

Hence, proved $\overline{A+B} = \bar{A} \cdot \bar{B}$

De Morgan Applications—

De Morgan's theorem is useful in the implementation of the basic gate operations with alternative gates, particularly with NAND and NOR gates which are readily available in IC form.