

Ans 2 $f(x) = \log\left(\frac{1-x}{1+x}\right)$

Put $x = y$

$$f(y) = \log\left(\frac{1-y}{1+y}\right)$$

$$\text{L.H.S} = f(x) + f(y) = \log\left(\frac{1-x}{1+x}\right) + \log\left(\frac{1-y}{1+y}\right)$$

$$= \log\left[\frac{(1-x)}{(1+x)} \cdot \frac{(1-y)}{(1+y)}\right]$$

$$= \log \left(\frac{(1-x-y+xy)(R+R-1)}{R+R+x+1} \right)$$

$$= \log \left[\frac{(1+xy)(R+R) - (x+y)}{(1+xy) + (R+R)} \right] \quad \text{--- (i)}$$

R.H.S = $f \left(\frac{x+y}{1-xy} \right) = f(t)$

$$t = \log \left(\frac{1-t}{1+t} \right)$$

$$= \log \left(\frac{1 - \frac{x+y}{1-xy}}{1 + \frac{x+y}{1-xy}} \right)$$

$$= \log \left[\frac{(1-xy) - (x+y)}{(1-xy) + (x+y)} \right]$$

$$= \log \left[\frac{(1-xy) - (x+y)}{(1-xy) + (x+y)} \right] \quad \text{--- (ii)}$$

(i) & (ii) are equal

$$f(x) + f(y) = f \left(\frac{x+y}{1-xy} \right)$$