$\sqrt{l^2+m^2+n^2}=1$

$l^2+m^2+n^2=1$

Section-6

Ans-1 Let $f: X \to Y$ be a funcation and $A, B \subseteq X$

Prove $f(A \cup B) = f(A) \cup f(B)$

Note $f(A) = \{y \in Y \mid y = f(x) \text{ for some } x \in A\}$

Proof → claim $f(A \cup B) = f(A) \cup f(B)$

Date
Page

Take any $y \in f(A \cup B)$ This means $\exists x \in A \cup B$ s.t $f(x) = y$ since $x \in A \cup B$, then $x \in A$ or $x \in B$, so

$y = f(x) \in f(A)$ or $y = f(x) \in f(B)$, so $y \in f(A) \cup f(B)$ Thus $f(A \cup B) \subseteq f(A) \cup f(B)$.

Take any $y \in f(A) \cup f(B)$, so $y \in f(A)$ or $y \in f(B)$

Assume wlog $y \in f(A)$, This means $\exists x \in A$ s.t. $y = f(x)$ Now $A \subseteq A \cup B$, so $x \in A \cup B$, so $y = f(x) \in f(A \cup B)$ This shows $f(A) \cup f(B) \subseteq f(A \cup B)$.

$\therefore f(A \cup B) = f(A) \cup f(B)$. Ans