

$$\sqrt{l^2 + m^2 + n^2} = 1$$

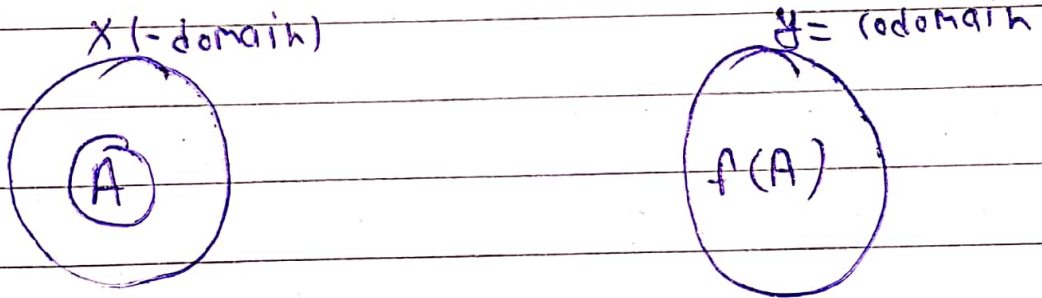
$$l^2 + m^2 + n^2 = 1$$

Section-6

Ans-1 Let $f: X \rightarrow Y$ be a function and $A, B \subseteq X$

$$\text{Prove } f(A \cup B) = f(A) \cup f(B)$$

Note $f(A) = \{y \in Y \mid y = f(x) \text{ for some } x \in A\}$



Proof \rightarrow claim $f(A \cup B) = f(A) \cup f(B)$

Take any $y \in f(A \cup B)$ This means $\exists x \in A \cup B$
 s.t. $f(x) = y$ since $x \in A \cup B$, then $x \in A$ or
 $x \in B$, so

$y = f(x) \in f(A)$ or $y = f(x) \in f(B)$, so
 $y \in f(A) \cup f(B)$ Thus $f(A \cup B) \subseteq$
 $f(A) \cup f(B)$.

Take any $y \in f(A) \cup f(B)$, so $y \in f(A)$ or $y \in f(B)$

Assume wlog $y \in f(A)$, This means $\exists x \in A$
 s.t. $y = f(x)$ Now $A \subseteq A \cup B$, so $x \in A \cup B$, so
 $y = f(x) \in f(A \cup B)$ This shows
 $f(A) \cup f(B) \subseteq f(A \cup B)$.

$\therefore f(A \cup B) = f(A) \cup f(B)$. \square