

Sec-2: ans 1 ⇒ Yield Criterion :-

It is a hypothesis concerning the limit of elasticity under any possible combination of stresses. Several yield criteria have been proposed to find out the relation b/w yield criteria have been proposed to find out the relation b/w yield stresses.

⇒ Tresca's Theory :-

1. It states that failure of a material (as plastic deformation) will occur when the maximum shear stress in a material reaches its value of maximum shear stress at elastic limit.
2. Let principal stresses at a point in the material are σ_1 , σ_2 and σ_3 ($\sigma_1 > \sigma_2 > \sigma_3$)
So maximum shear stress is given by.
$$\frac{\sigma_1 - \sigma_3}{\sigma_2} = \tau_{\max}$$
3. Plastic deformation occurs when τ_{\max} is equal to K ($K = \text{max. shear stress at elastic limit}$). So acc. to Tresca's theory of plastic deformation :-
$$\frac{\sigma_1 - \sigma_3}{2} = K \quad \text{--- (1)}$$
4. for uniaxial tension condition,
$$\sigma_1 = \sigma_y$$
$$\sigma_2 = \sigma_3 = 0$$

5. for Uniaxial compression condition,

$$\sigma_1 = \sigma_2 = 0$$

$$\sigma_3 = -\sigma_y$$

6. for Uniaxial tension condition, put the value of σ_1 and σ_3 in eq ①

$$\frac{\sigma_y - 0}{2} = K$$

$$K = \frac{\sigma_y}{2}$$

7. for Uniaxial compression condition, put the value of σ_1 and σ_3 in eq ①.

$$\frac{0 - (-\sigma_y)}{2} = K$$

$$K = \frac{\sigma_y}{2}$$

→ Von Mises Theory :-

1. It states that failure of a material will occur when the total shear strain energy per unit volume in strained material reaches a value equal to the shear strain energy per unit volume at the elastic limit.

2. Energy of distortion is given as :-

$$U = \frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

Here, U is the shear strain energy per unit volume, σ_1 , σ_2 and σ_3 are the principal stresses, G is the shear modulus.

3. So, $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 126 \times 10^6 = C$
where, $C = \text{constant}$.

4. for uniaxial tensile loading,

$$\sigma_1 = \sigma_y, \quad \sigma_2 = \sigma_3 = 0$$

Putting its value in above eqⁿ :-

$$(\sigma_y - 0)^2 + (0 - 0)^2 + (0 - \sigma_y)^2 = C$$

$$2\sigma_y^2 = C$$

5. Considering yielding under pure tension,
for pure shear, $\sigma_1 = K, \sigma_2 = 0, \sigma_3 = -K$

$$\therefore (K - 0)^2 + (0 - (-K))^2 + (-K - K)^2 = C$$

$$6K^2 = C$$

$$2\sigma_y^2 = 6K^2$$

$$K = \frac{\sigma_y}{\sqrt{3}}$$