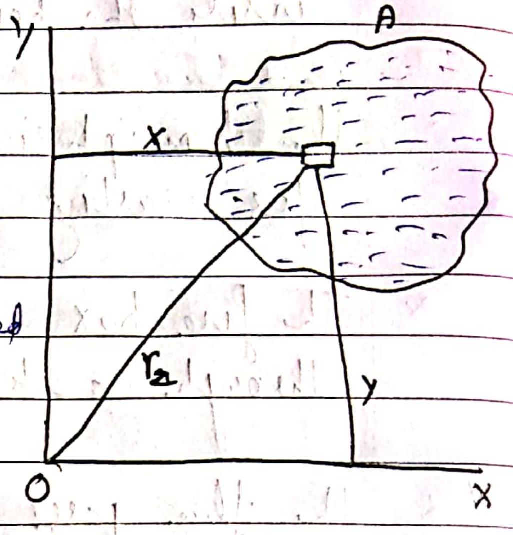
Sec I. ans - 1 => Product of Inertia :-

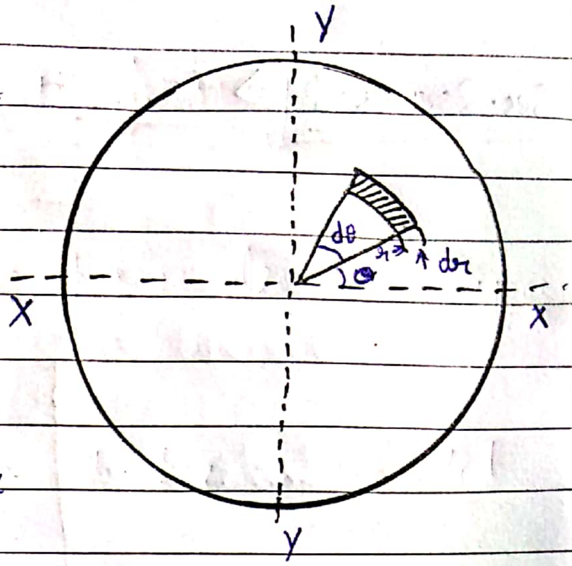


The product of inertia of area A relative to the indicated XY regulated axes is $I_{XY} = \int xy\, dA$. The product of inertia of the mass contained in volume V relative to the XY axes is $I_{XY} = \int xy \rho\, dV$ — Similarly for I_{YZ} and I_{ZX}.

Relative to principal axes of inertia, the product of inertia of a fig. is Zero. If a fig. is mirror symmetrical about YZ plane,
$I_{ZX} = I_{XY} = 0$

- Product of inertia of a body are measure of symmetry.
- If a particular plane is a plane of symmetry, then the product of inertia associated with any axis perpendicular to that plane are zero.
- For example, consider a thin laminate. The mid-plane of the laminate lies in the XY-plane so that it thickness is above the plane and half is below. Hence, XY-plane is a plane of symmetry and $I_{XY} = I_{YZ} = 0$.
- Product of inertia are found either by measurement or calculation. Calculation are based on direct integration or on the "body build-up" Technique. In the body built-up technique, product of inertia of simple shapes are added to estimate the product of inertia of a composite shape.

1. Consider an elemental area $r\,d\theta$ & thickness dr as shown in fig.



Mass of element, $dm = \rho r\,d\theta\,dr\,t$
$= \rho t\,r\,d\theta\,dr$

where, ρ = density of the circular plate.
t = thickness of the plate

Its distance from X-axis $= r\sin\theta$

2. Now,
$$I_{xx} = \oint (r\sin\theta)^2\,dm$$
$$= \int_0^R\int_0^{2\pi} r^2 \sin^2\theta\,\rho t\,r\,d\theta\,dr$$
$$= \rho t\int_0^R\int_0^{2\pi} r^3\left(\frac{1-\cos 2\theta}{2}\right)dr\,d\theta$$
$$= \rho t\int_0^R \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr = \rho t\int_0^R \frac{r^3}{2}\times 2\pi\,dr$$
$$= \rho t\pi\left[\frac{r^4}{4}\right]_0^R = \rho t\,\frac{\pi R^4}{4}$$

3. Mass of the plate, $M = \rho\times\pi R^2 t$
$$I_{xx} = MR^2/4$$
Similarly, $I_{yy} = MR^2/4$

Actually $I = \frac{MR^2}{4}$ is the moment of inertia of circular plate about any diametral axis in the plate.

4. But total Mass, $M = \rho t\pi R^2$
$$I_{zz} = \frac{MR^2}{2}$$