Section – 5

Q3

Ans → Since R and G are negligible, transmission line equation becomes

$$\frac{\partial e}{\partial x} = -L \frac{\partial i}{\partial t} \quad \text{— (1)}$$

and

$$\frac{\partial i}{\partial x} = -C \frac{\partial e}{\partial t} \quad \text{— (2)}$$

For elimination of i, differentiating eqⁿ (1) partially w.r.t 'x' and eqⁿ (2) partially w.r.t 't' we get :—

$$\frac{\partial^2 e}{\partial x^2} = -L \frac{\partial^2 i}{\partial x \partial t}$$

$$\frac{\partial^2 i}{\partial t \partial x} = C \frac{\partial^2 e}{\partial t^2}$$

hence,

$$\frac{\partial^2 e}{\partial x^2} = LC \frac{\partial^2 e}{\partial t^2} \quad \text{— (3)}$$

- The initial conditions are $i(x,0) = i_0$,
  $$e(x,0) = e_0 \sin \frac{\pi x}{l} \quad \text{— (4)}$$
- Since, the ends are suddenly grounded, the boundary conditions are
  $$e(0,t) = e(l,t) = 0 \quad \text{— (5)}$$

Also,

$i = i_0$ (constant) when $t = 0$

$\frac{\partial i}{\partial x} = 0$ which gives $\frac{\partial e}{\partial t} = 0$ when $t = 0$ — (6)

Cond

- Note let $e = XT$ be a sol^n of eq^n (3) where,

X is a function of x only &
T is a function of t only.

$$\frac{\partial^2 e}{\partial x^2} = X''T \quad \&$$
$$\frac{\partial^2 e}{\partial t^2} = XT''$$

$\therefore$ from eq^n (3) $X''T = LC\, XT''$

Separating the variable $\frac{X''}{X} = LC \frac{T''}{T} = -p^2$ (say) unII

- This leads to the ordinary differential equations.

$$\frac{d^2X}{dx^2} + p^2 X = 0 \quad \&$$
$$\frac{d^2T}{dt^2} + \frac{p^3}{LC} T = 0$$

$\therefore \quad X = C_1 \cos px + C_2 \sin px$

$$T = C_3 \cos \frac{pt}{\sqrt{LC}} + C_4 \sin \frac{pt}{\sqrt{LC}}$$

$$e = XT = (C_1 \cos px + C_2 \sin px)\left(C_3 \cos \frac{pt}{\sqrt{LC}} + C_4 \sin \frac{pt}{\sqrt{LC}}\right) \quad \text{--- (7)}$$

$\Rightarrow$ Applying the boundary conditions eqn. (5) in eqn. (7), we get

$C_1 = 0$ & $p = \frac{n\pi}{l}$, n being an integer.

$\therefore$ Eqn. (7) becomes.

$$e = C_2 \sin \frac{n\pi x}{l} \left(C_3 \cos \frac{n\pi t}{l(\sqrt{LC})} + C_4 \sin \frac{n\pi t}{l\sqrt{LC}}\right)$$

or

$$e = \sin \frac{n\pi x}{l} \left(A \cos \frac{n\pi t}{l\sqrt{LC}} + B \sin \frac{n\pi t}{l\sqrt{LC}}\right) \quad \text{--- (8)}$$

where

$A = C_2 C_3$ and
$B = C_2 C_4$

$$\frac{\partial e}{\partial t} = \sin\frac{n\pi x}{l}\left(-\frac{An\pi}{l\sqrt{LC}}\sin\frac{n\pi t}{l\sqrt{LC}} + \frac{Bn\pi}{l\sqrt{LC}}\cos\frac{n\pi t}{l\sqrt{LC}}\right)$$

Since $\frac{\partial e}{\partial t} = 0$ when $t = 0$, we get

$$B = 0$$

$\therefore$ from eqn (8)

$$e = A\sin\frac{n\pi x}{l}\cdot\cos\frac{n\pi x}{l\sqrt{LC}}$$

By superposition,

$$e = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{l}\cdot\cos\frac{n\pi x}{l\sqrt{LC}}$$

is also a solution.

But,

$$e = e_0 \sin\frac{\pi x}{l}, \text{ when } t = 0$$

$$\therefore \quad e_0 \sin\frac{\pi x}{l} = \sum_{n=1}^{\infty} A_n \sin\frac{n\pi x}{l}$$

$$\Rightarrow \quad A_1 = e_0 \;\&\; A_2 = A_3 = \cdots\cdots = 0$$

Hence,

$$e = e_0 \sin\frac{\pi x}{l}\cdot\cos\frac{\pi t}{l\sqrt{LC}}$$

Now,

$$-L\frac{\partial i}{\partial t} = \frac{\partial e}{\partial x}$$

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$$\therefore \frac{\partial i}{\partial t} = -\frac{1}{L} \cdot \frac{e_0 \pi}{l} \cdot \cos\frac{\pi x}{l} \cdot \cos\frac{\pi t}{l\sqrt{LC}}$$

• Integrating w.r.t 't', regarding 'x' as constant.

$$i = \frac{e_0 \pi}{L l} \cdot \cos\frac{\pi x}{l} \cdot \frac{l\sqrt{LC}}{\pi} \cdot \sin\frac{\pi t}{l\sqrt{LC}} + f(x) \quad \text{--- (9)}$$

where $f(x)$ is an arbitrary constant function.

Since $i = i_0$ when $t = 0$, we have $i_0 = 0 + f(x)$ or

$$f(x) = i_0$$

$\therefore$ from eqn. (9), we get

$$\boxed{i = i_0 - e_0\sqrt{\frac{C}{L}} \cos\frac{\pi x}{l} \cdot \sin\frac{\pi t}{l\sqrt{LC}}}$$

Ans.