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Section-5

Q1

Ans -> Laplace equation in two dimension is given by :-

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0 \quad \text{--- (1)}$$

Let,

$$u = XY \quad \text{--- (2)}$$

then,

$$\frac{\partial^2 u}{\partial x^2} = X''Y$$

&

$$\frac{\partial^2 u}{\partial y^2} = X.Y''$$

Substituting in eqn (1), we get

$$X''Y + X.Y'' = 0$$

or

$$\frac{X''}{X} = -\frac{Y''}{Y} = K \text{ (say)} \quad \text{--- (3)}$$

Now,

$$\frac{d^2X}{dx^2} - KX = 0 \quad \text{--- (4)}$$

$$\frac{d^2Y}{dy^2} + KY = 0$$

solving eqn (4), we get

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(i) When k is positive and $k = p^2$

$$X = C_1 e^{px} + C_2 e^{-px}$$
$$Y = C_3 \cos py + C_4 \sin py$$

(ii) When k is negative and $k = -p^2$

$$X = C_1 \cos px + C_2 \sin px$$
$$y = C_3 e^{py} + C_4 e^{-py}$$

(iii) When $k = 0$

$$X = C_1 x + C_2$$
$$Y = C_3 y + C_4$$

Thus, the various possible solution of Laplace eq^n. (2) are

$$u = (C_1 e^{px} + C_2 e^{-px})(C_3 \cos py + C_4 \sin py) \quad \text{--- (5)}$$

$$u = (C_1 \cos px + C_2 \sin px)(C_3 e^{py} + C_4 e^{-py}) \quad \text{--- (6)}$$

$$u = (C_1 x + C_2)(C_3 y + C_4) \quad \text{--- (7)}$$

from these three solutions, we have to choose that solution which is consistent with the physical nature of the problem & the given boundary conditions.

Ans.