Section-2

Q3

Ans - (a) $(x^2 - yz)p + (y^2 - zx)q = z^2 - xy.$

Here Lagrange's subsidiary eq^n. are :-

$$\frac{dx}{x^2 - yz} = \frac{dy}{y^2 - zx} = \frac{dz}{z^2 - xy}$$

$$\therefore \frac{dx - dy}{(x-y)(x+y+z)} = \frac{dy - dz}{(y-z)(x+y+z)} = \frac{dz - dx}{(z-x)(x+y+z)}$$

taking the first two members, we have

$$\frac{dx - dy}{x-y} = \frac{dy - dz}{y-z}$$

which on integration gives :-

$$\log(x-y) = \log(y-z) + \log a$$

or $\log\left(\frac{x-y}{y-z}\right) = \log a$ or $\frac{(x-y)}{(y-z)} = a$ — (1)

Similarly, taking the last two members, we get

$$\frac{(y-z)}{(z-x)} = b$$

Scanned with CamScanner

from eq. (1) and eqⁿ (2) the general solution is

$$\boxed{\phi\left(\frac{x-y}{y-z}, \frac{y-z}{z-x}\right) = 0}$$

Ans.

(b) $y^2 \frac{z}{x} p + xzq = y^2$

- Rewriting the given eqⁿ. as :-

$$x^2 zp + x^2 zq = y^2 x$$

- The subsidiary eqⁿ. are :-

$$\frac{dx}{y^2 z} = \frac{dy}{x^2 z} = \frac{dz}{y^2 x}$$

The first two fractions give, $x^2\,dx = y^2\,dy$.

Integrating, we get

$$x^3 - y^3 = a \quad \text{— (1)}$$

again the first & third fractions give $x\,dx = z\,dz$

Integrating, we get

$$x^2 - z^2 = b \quad \text{— (2)}$$

Hence from eq^n (1) & eq^n (2), the complete solution is,

$$\boxed{x^3 - y^3 = f(x^2 - y^2)}$$

Ans