Section - 2

Q2

Ans. → $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Given, $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

Let

$$u = XY$$

$$Y \frac{\partial^2 X}{\partial x^2} + X \cdot \frac{\partial^2 Y}{\partial y^2} = 0$$

$$\frac{1}{X} \cdot \frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} = -k^2 \text{ (say)}$$

$$X = c_1 \cos kx + c_2 \sin kx$$
$$Y = c_3 e^{ky} + c_4 e^{-ky}$$

So,

$$u = (c_1 \cos kx + c_2 \sin kx)(c_3 e^{ky} + c_4 e^{-ky}) \quad \text{— (1)}$$

$$u(0,y) = 0$$

$$0 = c_1 (c_3 \cdot e^{ky} + c_4 \cdot e^{-ky})$$

$$\boxed{c_1 = 0}$$

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from eq^n ①, we get

$$u = \sin kx \left(A_n e^{ky} + B_n \cdot e^{-ky}\right) \quad \text{— ②}$$

$$u(\pi, y) = 0$$

$$\sin(k\pi) = 0 \Rightarrow k = n$$

$$u = \sin nx \left(A_n e^{ny} + B_n e^{-ny}\right) \quad \text{— ③}$$

$\lim_{y \to \infty} u(x,y) = 0$, it satisfies only when $A_n = 0$

from eq^n. ③, we get

$$u = \sum B_n \cdot e^{-ny} \sin nx \quad \text{— ④}$$

Now,

$$u(x, 0) = u_0$$

$$u_0 = \sum B_n \cdot \sin nx$$

$$B_n = \frac{2}{\pi} \int_0^{\pi} u_0 \sin nx \, dx = -\frac{2u_0}{\pi} \left[\frac{\cos nx}{n}\right]_0^{\pi}$$

$$= \frac{-2u_0}{\pi} \left[\frac{(-1)^n - 1}{n}\right]$$

thus from eq^n. ④

$$\boxed{u = \sum -\frac{2u_0}{\pi n} \left[(-1)^n - 1\right] e^{-ny} \sin nx}$$

Ans.